# Modelling the collapse of complex societies


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## Easter Island model

We propose the following model for the dynamics of the population $x$, resources $y$ and material goods $z$ on Easter Island:

$$\begin{aligned} \dot{x} &= (b - de^{-z/(\rho x)})x \\ \dot{y} &= ry(1 - y/K) - \alpha xy \\ \dot{z} &= \alpha xy - sx(1 - e^{-z/(\rho x)}) - cz \end{aligned} \tag{1}$$

The model has been proposed in [1], with parameters values consistent with the model by [2] while also matching the archaeological record [3], see Figs. 1 and 2. We couple via diffusion two systems described by (1) with different extraction rates $\alpha_1, \alpha_2$ and see how the this effects the equilibrium state, see Fig. 3.

## Easter Island data and model fit to population and resources

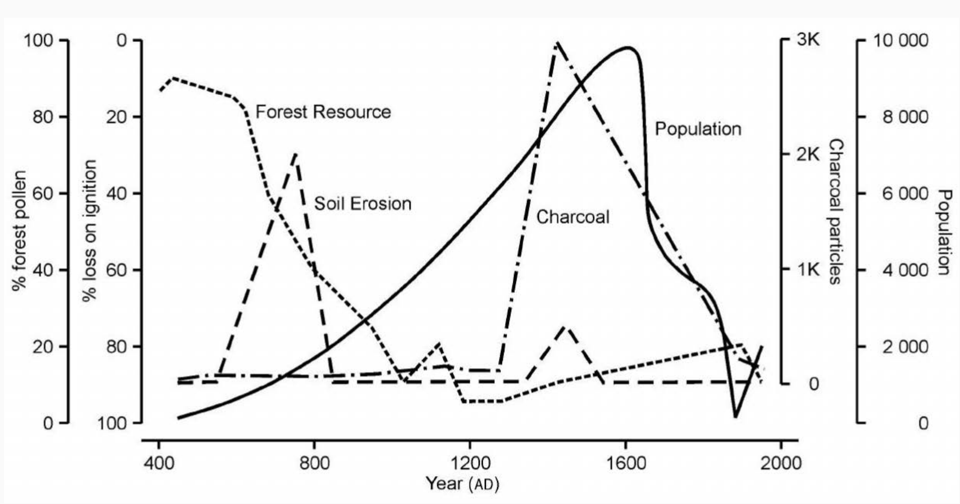


Fig. 1: Archaeological record of Easter Island.

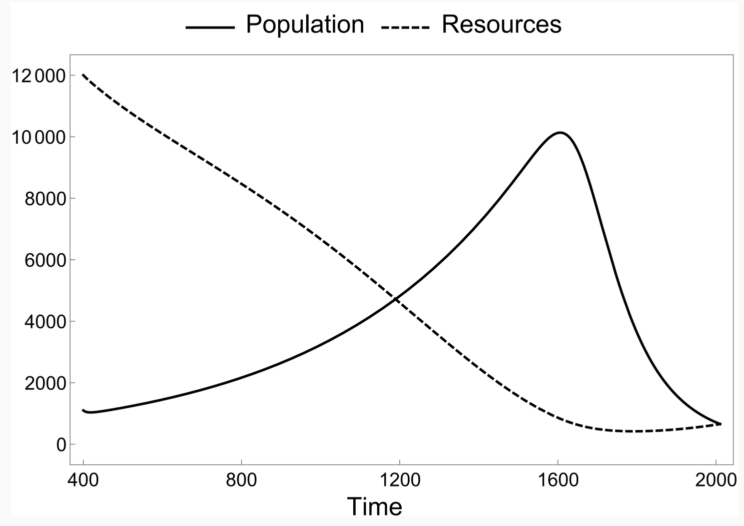


Fig. 2: Population and resources as determined by model (1).

## Maya civilisation model

Let $x_s, x_i$ and $x_b$ be the population involved in swidden agriculture, intensive agriculture and monument building, $y$ the resources and $z$ the monuments built over time. We propose the following system for the dynamics of the Maya socio-environmental system:

$$\begin{aligned} \dot{x}_s &= [(1-\tau) + \tau p_s]\beta n x - \beta n^{-\delta} x_s + \sigma[(1-\theta(n))x_b - \theta(n)x_s] \\ \dot{x}_i &= \qquad \tau p_i \beta n x - \beta n^{-\delta} x_i \\ \dot{x}_b &= \qquad\quad -\beta n^{-\delta} x_b - \sigma[(1-\theta(n))x_b - \theta(n)x_s] \\ \dot{y} &= w_t r y(1 - y/(w_t K)) - sdnx \\ \dot{z} &= bx_b - mz \end{aligned} \tag{2}$$

where $x$ and $n$ are given by: $x = x_s + x_i + x_b$

$$n = w_t \frac{x_s + \alpha x_i}{x_s + x_i + x_b}(1 - e^{-y/(w_t c K)})$$

and $\theta(n)$ is a step function at $n = 2$.

## Maya population and the rate of monument building

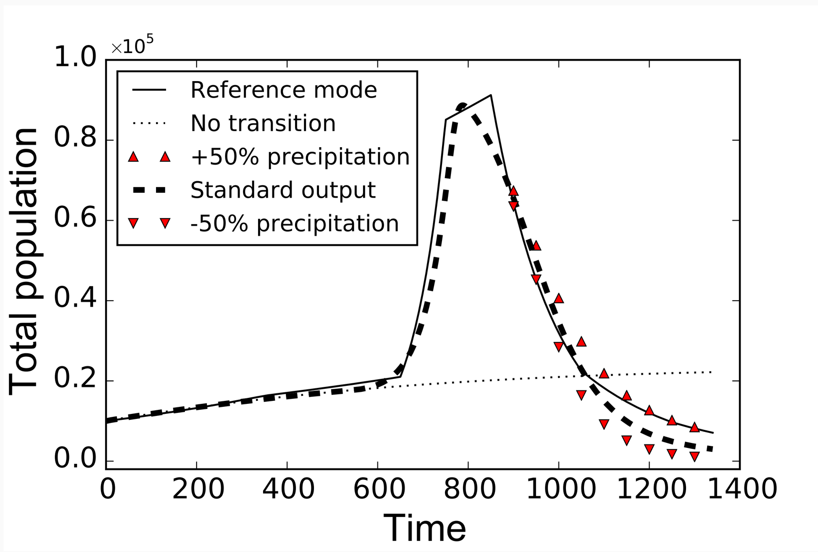


Fig. 3: Model predictions for the total population when precipitation is: normal, enhanced by 50% and reduced by 50%.

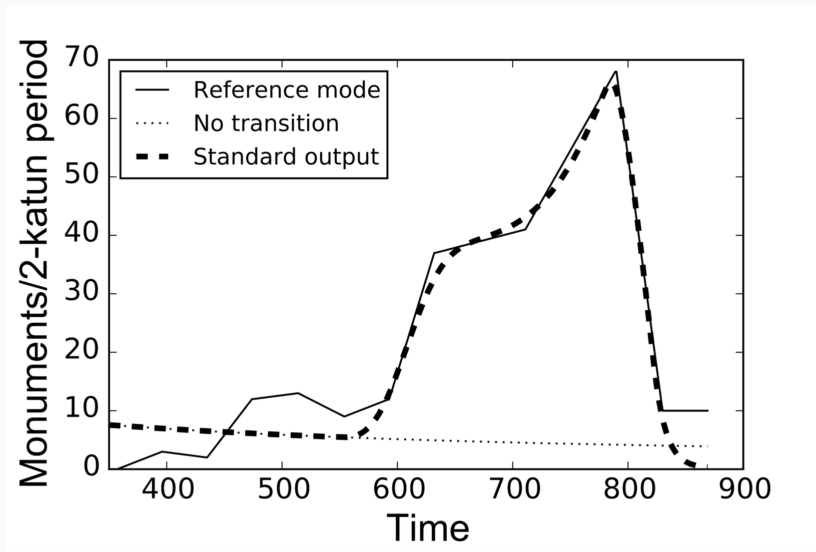


Fig. 4: Model predictions for monument building rates in the case of normal precipitation with and without agricultural transition.

## Roman Empire model

Let $x$ be the army size, $y$ the land conquered, and $z_1$ the amount of silver used in minting (measured in coins) and $z_2$ the number of minted coins. The model is:

$$\begin{aligned} \dot{x} &= p_0\left(\frac{z_1}{p_1 z_2} - 1\right) - \lambda\delta(t - t_d)x \\ \dot{y} &= p_2\left(\frac{x}{p_3} - 1\right) + p_4\left(\frac{z_1}{p_1 z_2} - 1\right) - \lambda\delta(t - t_d)y \\ \left(\frac{z_1}{z_2}\right)' &= p_5\left(1 - \frac{y}{p_6}\right) \end{aligned} \tag{3}$$

where the parameters $p_0, \cdots, p_7$ are assumed non-negative and determined such that the trajectories the system generates match as closely as possible to the reference modes.

## Roman Empire territorial extent and salvation policy

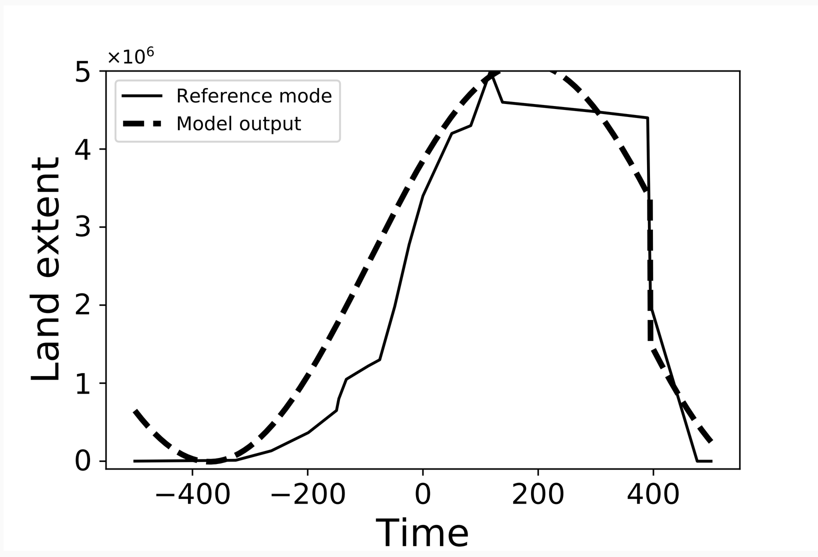


Fig. 5: Historical reference modes (solid lines) with model output (dashed lines) for the land extent.

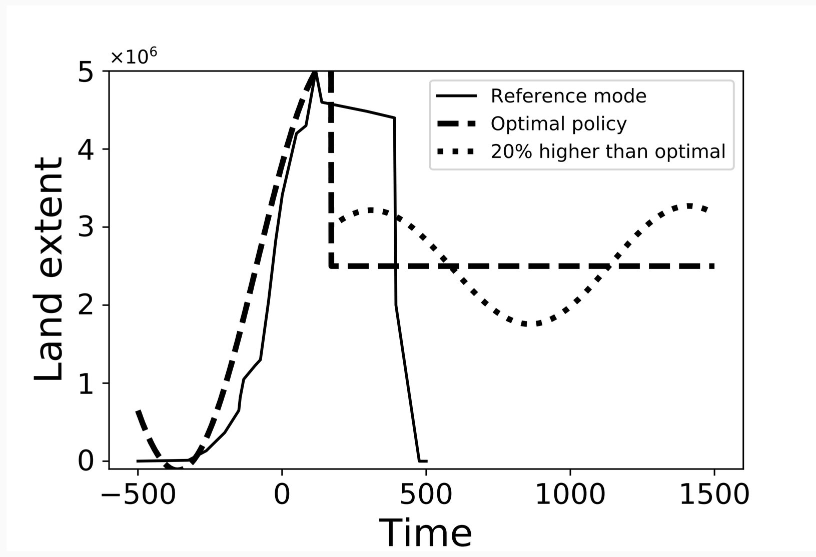


Fig. 6: Optimal policy (dashed lines) and another where stock values are 20% higher than the optimum (dotted lines).

## General model

A network model of agrarian societies:

$$\begin{aligned} \dot{x}_i &= \left(b\frac{z_i}{x_i + \epsilon} - dx_i\right)(1 - e^{-z_i/\epsilon}) - \sigma\sum_{j=1}^{N} x_j B_{ij} \\ \dot{y}_i &= ry_i\left(1 - \frac{y_i}{K}\right) - \alpha_i x_i y_i \sum_{j=1}^{N} x_j A_{ij} \\ \dot{z}_i &= \alpha_i x_i y_i \sum_{j=1}^{N} x_j A_{ij} - cz_i \end{aligned} \tag{4}$$

where each region $i$ has population $x_i$, natural resources $y_i$, capital $z_i$.

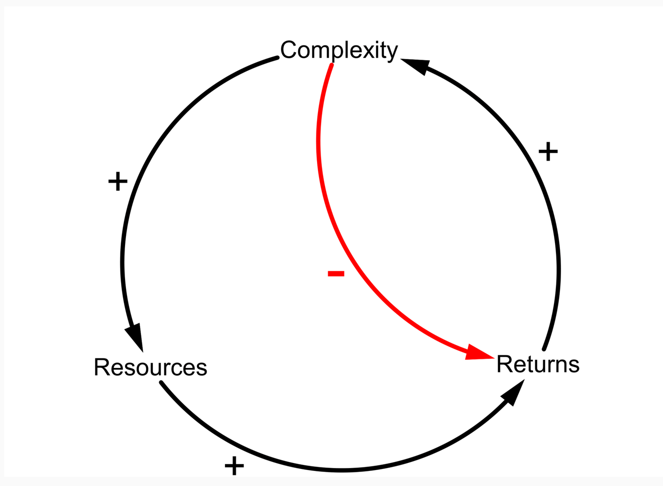


Fig. 7: General feedback mechanism.

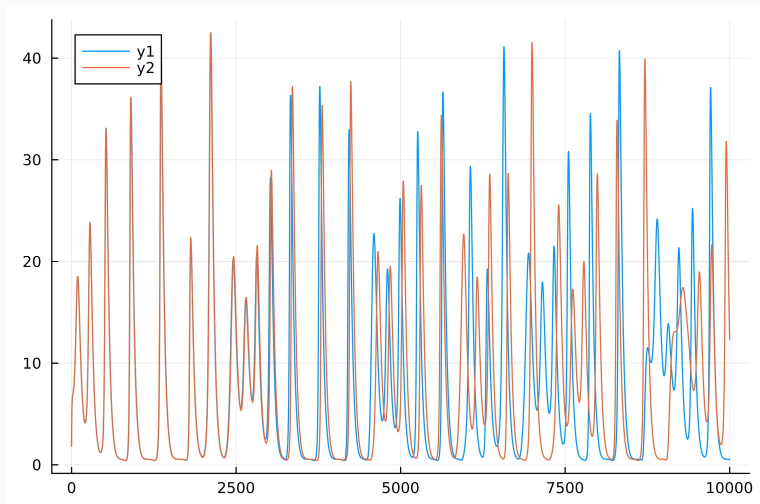


Fig. 8: Chaotic behavior for large extraction rate $\alpha$.

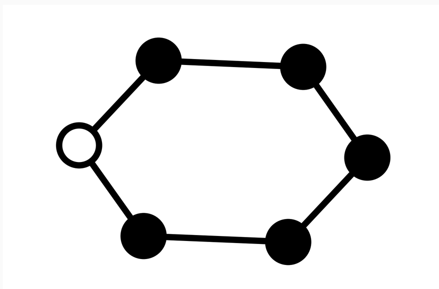

Fig. 9: Ring topology, each node is a region connected via diffusion. White is sustainable, black is unsustainable.

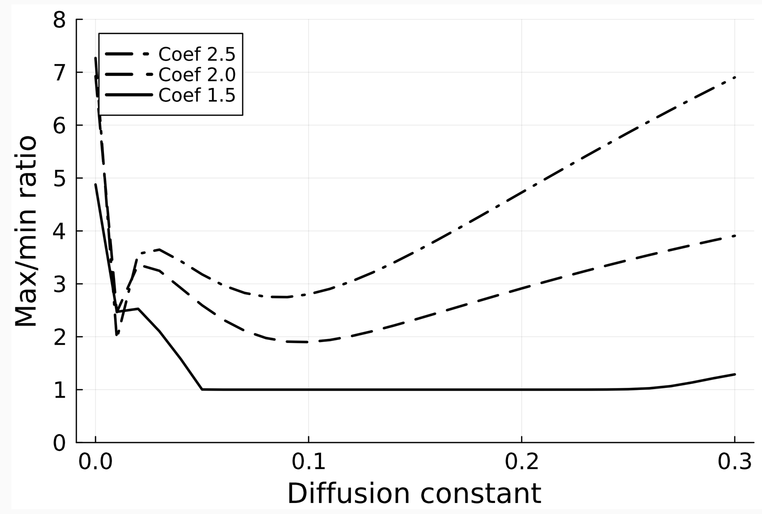


Fig. 10: The ratio between the maximum and minimum total population for different values of the extraction rate.

## Timescales

| Phenomena / Social Forces | Timescale (years) |
|---|---|
| Biological evolution, toolmaking, climate change | 30,000–100,000 |
| Agriculture, domestication, power hierarchies | 3,000–10,000 |
| Religion, technology, complexity feedback loops | 300–1,000 |
| Dynastic policies, royal traditions | 30–100 |
| Economic cycles, war, pandemics, politics | 3–10 |

## References


[1] Roman, S., Bullock, S. & Brede, M. Coupled Societies are More Robust Against Collapse: A Hypothetical Look at Easter Island. Ecological economics, 132:264-278, 2017.
[2] Roman, S., Palmer, E., & Brede, M. (2018). The dynamics of human–environment interactions in the collapse of the Classic Maya. Ecological economics, 146, 312-324.
[3] Roman, S. & Palmer, E. (2019). The growth and decline of the Western Roman Empire: quantifying the dynamics of army size, territory, and coinage. Cliodynamics, 10(2).
[4] Roman, S., & Bertolotti, F. (2023). Global history, the emergence of chaos and inducing sustainability in networks of socio-ecological systems. Plos one, 18(11), e0293391.



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