Power System Static-State Estimation,  
Part I: Exact Model  

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Abstract—The static state of an electric power system is defined as the vector of the voltage magnitudes and angles at all network buses. The static-state estimator is a data processing algorithm for converting redundant meter readings and other available information into an estimate of the static-state vector. Discussions center on the general nature of the problem, mathematical modeling, an interactive technique for calculating the state estimate, and concepts underlying the detection and identification of modeling errors. Problems of interconnected systems are considered. Results of some initial computer simulation tests are discussed.

INTRODUCTION

A REAL-TIME central control system can be used to improve the security (reliability) of electrical power system bulk generation and an EHV transmission system. It is possible to consider the operation of such a central control system in two steps: 1) raw information is processed in real time by a digital computer into a more useful form; and 2) control decisions are made from the processed information either by the digital computer or by a human operator. In this paper only certain aspects of the first or information processing stage are discussed. In particular, real-time digital computer programs (algorithms) for converting the available information (direct meter readings plus other information) into an estimate of the present state of the power system are discussed. Only the quasi-steady state or static operating behavior of the power system is considered. The vector whose elements are the voltage magnitudes and angles at all the generation and load buses is called the state vector of the system. Because of the quasi-steady-state assumption, this vector is called the static-state vector.

The subsequent use of the state estimate as an input to computer control programs or to drive displays for the system operator is not discussed. However, it should be clear that no control system (man or computer) can effectively tell a power system where to go in the future without some knowledge of its present state.

Static-state estimation is related to conventional load-flow calculations. However, the state estimator is designed to handle the many uncertainties associated with trying to do an on-line load flow for an actual system using meter readings telemetered in real time to a digital computer. Uncertainties arise because of meter and communication errors, incomplete metering, errors in mathematical models, unexpected system changes, etc. These uncertainties make for large differences between the usual load-flow studies done in the office for system planning and on-line estimation done as part of a control system.

The static-state estimator is based on classical mathematical techniques such as estimation, detection, probability, statistics, and filtering. However, an attempt is made to present the ideas without drawing on an extensive background in these theories. A more theoretical presentation would have had some advantages, but the chosen approach hopefully makes the concepts more widely accessible. Most of the specialized mathematical jargon is relegated to background discussions which can be skipped.

This paper is the first of a three-part series on the static-state estimator. In this paper, the general problem, model, and theory of solution are developed. In [2] approximate models and solutions are developed. In [3] consideration is given to implementation problems related to computational speed, dimensionality of the state vector, and the fact that a power system is never truly in steady state.

The static-state estimator results from a combination of two big fields, load flow, and statistical estimation theory so no attempt will be made to provide a complete bibliography. In [4] a clear, readily available load-flow reference with an extensive bibliography is provided. Basic statistics references are given in [1], [6]. However, there are many other good papers and books which also contain the same related material. Material is contained in [7] that is very similar to the static-state estimator concept; differences lie primarily in details and emphasis.

The following notational conventions are used. A bold face letter denotes a vector or matrix. All vectors are column vectors. A prime denotes matrix transpose and $-1$ denotes matrix inverse. The letter $E$ denotes the expectation or averaging operation on a random variable or vector. Complex numbers or matrices are denoted by a tilde; i.e., $\tilde{Y}$ is a real number while $\tilde{Y}$ is a complex number.

NATURE OF PROBLEM

A power system rarely achieves a true steady-state (static) operating point, as loads and generation patterns are continually changing. However, it is often a reasonable approximation to consider a power system to be in steady state over some short interval of time.

The power systems of much of the United States and Canada are interconnected. However, this paper is concerned with the state estimation problems associated with the central control of only some small portion of the overall power system. The term own system (OS) refers to that portion of the total power system of direct concern. OS is connected to the interconnected systems (IS) by tie lines. OS need not be a single utility; it may be a pool or region, etc.

Meters (watt, volt, var, etc.) can be placed on generator buses, EHV transmission lines, tie lines, and load buses, and the readings can be telemetered in real time to the central control system. However, real-time meter readings from the IS are not assumed to be available to OS. Furthermore, for the sake of generality, real-time measurements on all OS loads (stepdown transformer

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to a lower voltage distribution network) are not assumed to be available. Real-time measurements are assumed to be available only on OS generators, some EHV transmission lines, and all tie lines. Situations where more real-time metering is available are special cases or require only simple modification. The real-time meter readings will not be perfect. Small random errors are always present. Bad data points will occur. Sometimes a meter reading will be lost completely.

The transmission network is modeled by a complex admittance matrix $\mathbf{Y}$. The vector $x$ is used to denote the voltage magnitudes and angles at all the buses (relative to some reference). The mathematical model based on $\mathbf{Y}$ is only an approximation to the actual system behavior.

OS control has to know the present network flows (voltage, power, current, etc.) on OS in order to predict the flows that will result if a transmission line or generator is removed (or added) in the near future (several hours). To estimate the present flows, it is reasonable first to estimate the state vector $x$ and then use $\mathbf{Y}$ and Kirchhoff's laws to obtain the desired estimated flows. The prediction problem is conceptually quite similar. Since the state vector $x$ contains the bus voltages, it will change if a line is removed. However, it is reasonable to assume that the generator real powers and voltage levels and the load real and reactive powers will not change (except during a brief transient). Thus an estimate of the state vector $x$ after the line is removed can be calculated from the present $x$, and hence the resulting flows can be predicted. A change in the generation-load pattern can be handled in the same way. If predicting many minutes or hours into the future is desired, load forecasting and knowledge of generator control laws must also be included. The problems of estimating the present flows and predicting future flows are conceptually similar, but in practice they are very different. To estimate present flows on OS, the state vector $x$ actually need contain only the voltages at the OS generators, loads, and tie lines. To predict the effect of changing the transmission network or generation pattern, the state vector $x$ must include buses of both IS and OS.

Since real-time meter measurements are assumed to be available only from meters on OS, more information must be added in the form of assumptions or data transferred in nonreal time from IS to OS. Similar additional information may also be needed on OS load buses that are not metered in real time.

**Modeling**

**Static Structure**

The network generator-load geometry and the network impedance values determine the static structure of a power system. One approach to modeling the static structure involves two concentric rings. The inner ring contains OS plus all systems with direct interconnections to OS and possibly others which may have a major effect on OS. The model to be used includes the static structures of only the systems in the inner ring. Each inner ring system, except OS, may have tie lines to outer ring systems. However, in the model all tie lines from any one inner-ring system to the outer-ring system are assumed to be tied to a single bus along with a single hypothetical generator which models the total interchange between the particular inner-ring system and all of the outer-ring systems. The transmission network of all the inner-ring systems is modeled by the one admittance matrix $\mathbf{Y}$. Of course, all network elements, generators, and loads are not explicitly included. Lower voltage distribution lines are ignored in calculating $\mathbf{Y}$, and small generator or load buses may be dropped or lumped together.

**Real-Time Meter Measurements**

Many meter measurements are transmitted to the central control system in real time. Let the vector $\mathbf{z}_{\text{meter}}$ denote the result of numbers fed into the computer. Let the vector $\mathbf{z}_{\text{ideal}}$ denote these numbers assuming no errors are made. Then

$$\mathbf{z}_{\text{meter}} = \mathbf{z}_{\text{ideal}} + \mathbf{z}_{\text{error}}$$

The vector $\mathbf{z}_{\text{error}}$ represents errors which come from many sources such as meter inaccuracies, communication errors, unbalanced phases, and the equivalent of analog to digital conversion. The error vector $\mathbf{z}_{\text{error}}$ is modeled as a zero mean random vector with $E[\mathbf{z}_{\text{error}}\mathbf{z}_{\text{error}}^\top] = \mathbf{0}_{\text{meter}}$, where $\mathbf{0}_{\text{meter}}$ is a positive definite, symmetric, matrix, the error covariance matrix. Let $\mathbf{z}_{\text{ideal}}$, model denote the vector of measurements that would be obtained if perfect meters were placed on the static-structure model. Since the static-structure model is not an exact representation of the actual system, $\mathbf{z}_{\text{ideal}}$ is not exactly equal to $\mathbf{z}_{\text{ideal}}$, model, but this difference is ignored under the assumption that other error sources dominate.

Let $x$ denote the state vector associated with the model, and let $x_{\text{true}}$, denote the true (but unknown) value of the state of the model (system). Then the real-time meter readings are modeled by

$$\mathbf{z}_{\text{meter}} = f(x_{\text{true}}) + \mathbf{z}_{\text{error}}$$

where $f(x)$ is a nonlinear function of $x$ determined by the admittance matrix $\mathbf{Y}$ and Kirchhoff's laws.

**Pseudomeasurements**

Real-time meter measurements do not contain all the available information on the power system. However, nonreal-time metered types of information are handled and modeled in the same manner as real meter measurements. Such pseudomeasurements are now discussed.

To illustrate the pseudomeasurement concept, consider a load bus whose real power demand is not sent to the central control system in real time. Assume it is known from records of past behavior that the load should be close to some nominal value of real power $P_{\text{nom}}$. This information is fed to the computer as a pseudomeasurement $z_1$ whose value is determined by $z_1 = P_{\text{nom}}$. Let $f_1(x)$ denote the real power at the load point that would be calculated from the static structure model. Then the pseudomeasurement $z_1$ is modeled as

$$z_1 = f_1(x_{\text{true}}) + \eta_1$$

where $\eta_1$ is a zero mean random variable with variance $\theta_1$. The value of $\theta_1$ is determined from past records; for example, as the average of the squares of the deviations of the actual real power is determined from the nominal value $P_{\text{nom}}$.

Some of the many types of nonreal-time metered information are now discussed. All of these can be incorporated into the model as pseudomeasurements with corresponding error variances. Certain load powers remain close to constant nominal values which can be treated as pseudomeasurements. Other loads vary widely during a day but remain close to some nominal percentage of the total load (or portion of load). Such percentages can be viewed as load-pattern coefficients which can be treated as pseudomeasurements. Nominal load-power factors can be treated as pseudomeasurements. If load-pattern coefficients are used, a total load daily cycle may also be needed which can be modeled as $P_{L,\text{total}}(t) = P_{\text{peak}}b(t)$, where $P_{\text{peak}}$ is the nominal daily peak and $b(t)$ (0 ≤ $t$ ≤ 24 hours) is the nominal daily load cycle.
Information on the nominal $P_{\text{peak}}$ can be treated as a pseudo-
measurement while $b(t)$ can be treated as being exact where actual 
deviations are accounted for by making the error variance 
associated with $P_{\text{peak}}$ sufficiently large. Some of the generators 
may be base loaded, and this base load output can be treated as a 
pseudo-measurement. Other generators will follow the load in a 
manner determined by control laws. Generator pattern coefficients 
can be based on the control laws to specify how the generation 
is distributed, and these coefficients can be treated as 
pseudo-measurements. Nominal real power sales between interconnected 
systems may be known and can be treated as pseudo-
measurements. Voltage levels at generators and some buses are often 
controlled, and the nominal values can be treated as pseudo-
measurements.

The actual pseudo-measurements to be used obviously depend 
on the type and amount of real meter measurements that are 
available. Thus generalizations are difficult. However, the follow-
ing list indicates one reasonable set of pseudo-measurements that 
is sufficient to yield a complete state estimate when combined 
with the meter measurements assumed to be available. This 
list is only an example. For OS pseudo-measurements are used for 
1) real power load-pattern coefficients and nominal values; 
2) nominal power factors at nonvoltage controlled load buses; 
and 3) nominal voltages at voltage controlled load buses. For 
each system in the inner ring, other than OS, pseudo-
measurements are used for 1) real power load-pattern coefficients or 
nominal values, 2) nominal power factors at load buses, 3) 
voltage magnitudes at generators, 4) real power generation 
distribution pattern coefficients or nominal values, 5) power sales 
to other companies in inner ring, 6) power sales to other com-
panies in outer ring, and 7) total daily load cycle. Obviously all 
the information on the IS must come from some place. It could 
be simply best guesses made by OS engineers. In a more ideal 
world, each inner-ring system would furnish OS the nominal values on a 
daily or weekly basis and instantaneously transmit to OS major 
unexpected changes as they occur. If best guesses are used, the 
associated error covariances are of course larger.

**Discussion of Model**

The static structure model is given by the admittance matrix 
$\tilde{Y}$ which includes only the important elements of the inner-ring 
systems. All real-time meter measurements and other types of 
information are modeled in the same way in terms of

$$z = f(x_{\text{true}}) + n$$

where $x$ is the state of the static structure model and $f(x)$ is 
determined by Kirchoff's laws and the admittance matrix $\tilde{Y}$. 
The error $n$ is modeled as a zero mean random vector with 
$E[nn'] = \Theta$.

In the discussions to follow on state estimation, the model 
(i.e., $\tilde{Y}, f(x)$) is assumed to be perfect. Obviously the model 
itself is often in error. Not all network elements are included in 
calculating $\tilde{Y}$. If a transmission line is unexpectedly lost, $\tilde{Y}$, 
and hence $f(x)$, is in error. The actual line admittances are not 
known perfectly. Modeling errors are also caused by bad data 
points resulting from meter, communication, or computer input 
failure (or worse, partial failure). Bad data points are viewed as 
an error in the modeling of $\Theta$ as they are measurements with 
errors much larger than assumed in the modeling of $\Theta$. Modeling 
errors are handled by detection and identification. Detection is 
concerned with spotting the existence of a modeling error while 
identification is concerned with locating and correcting the error.
meter. This reasonability argument for the choice of estimate can be supplemented by mathematical proofs if desired. For example, it can be shown that \( \hat{x} \) of (7) yields the smallest estimate error variance \( E[(x - x_{true})^2] \) for all unbiased estimates.

**State Estimation**

In the problem of interest (3), the dimensions of the vectors are larger, and \( f(x) \) is a nonlinear function of \( x \). Given \( x \), the state estimate \( \hat{x} \) is defined to be the value of \( x \) which minimizes

\[
J(x) = [z - f(x)]\theta^{-1}[z - f(x)].
\]

(9)

The minimization of \( J(x) \) of (9) with respect to \( x \) cannot be done in closed form as in the example because \( f(x) \) is a nonlinear function of \( x \). However, the following is a standard approach. Let \( x_0 \) be an initial guess for \( x \). Then a Taylor series expansion gives

\[
f(x) = f(x_0) + F(x_0)[\Delta x] + \cdots
\]

where \( F(x_0) \) is the Jacobian matrix whose elements are

\[
\frac{\partial f_{x_i}(x)}{\partial x_{x_j}} = x = x_0
\]

and \( \Delta x = x - x_0 \). If the higher order terms of (10) are neglected, substitution of (10) into (9) gives

\[
J(x) = [\Delta z - F(x_0)\Delta x]\theta^{-1}[\Delta z - F(x_0)\Delta x]
\]

(11)

where \( \Delta z = z - F(x_0) \). Equation 11 can be rewritten in the form

\[
J(x) = \Delta z[\theta^{-1} - \theta^{-1}F(x_0)\Sigma(x_0)F(x_0)\theta^{-1}][\Delta z] + [\Delta x - \Sigma(x_0)F(x_0)\theta^{-1}][\Delta z]
\]

(12)

where \( \Sigma(x_0) = [F(x_0)\theta^{-1}F(x_0)]^{-1} \) and \( \Sigma(x_0) \) is assumed to exist. Let \( \hat{\Delta x} \) denote the value of \( \Delta x \) that minimizes \( J(x) \) of (12). It is obvious from (12) that

\[
\Delta \hat{x} = \Sigma(x_0)F(x_0)\theta^{-1}[\Delta z]
\]

or

\[
\hat{x} = x_0 + \Sigma(x_0)F(x_0)\theta^{-1}[z - f(x_0)].
\]

(14)

If \( x_0 \) is close enough to \( \hat{x} \) to justify the dropping of the higher terms in (10), then (14) actually gives the desired \( \hat{x} \). In practice this is often not true so one defines a sequence \( x_n, n = 1, \ldots \) given by

\[
x_{n+1} = x_n + \Sigma(x_n)F(x_n)\theta^{-1}[z - f(x_n)].
\]

(15)

The iterations of (15) are continued until \( J(x) \) approaches a minimum. Possible stopping rules are to iterate until \( |J(x_{n+1} - J(x_{n+1})| \) or until the magnitude of all components of \( x_{n+1} - x_n \) are less than some predetermined value. Note that it is possible for \( J(x) \) to have local minimum and flat spots, and thus \( x_n \) may converge but not to \( \hat{x} \). It is also possible for \( x_n \) to never converge. If \( x_0 \) happens to be the true but unknown value \( x_{true} \), then it follows from (14) that

\[
E[(x_{true} - \hat{x})(x_{true} - \hat{x})^t] = \Sigma(\hat{x}).
\]

(16)

Equation (16) is actually an approximation valid for small errors. Thus \( \Sigma(x) \) can often be viewed as a measure of the accuracy of \( \hat{x} \).

The state estimate \( \hat{x} \) is defined as the value of \( x \) that minimizes \( J(x) \) of (9). This criterion has a long history of providing good estimates in a wide variety of problems (surveying, economics, biology, aerospace trajectories). Actually, the resulting \( \hat{x} \) can be shown to have a variety of mathematical optimum properties, and the criterion can be formally derived in various ways. If the errors are assumed to be Gaussian random variables, \( \hat{x} \) is a maximum likelihood estimate which has many nice properties, and the Cramer–Rao or information inequality can be used to interpret \( \Sigma \) [1], [6].

**Detection**

The state estimator algorithm of (15) is based on the assumption that the model (i.e., \( \hat{Y}(f(x), \theta) \)) is perfect. Detection of modeling errors is now discussed. Let \( \tilde{J} \) denote \( J(\hat{x}) \), i.e., \( J(x) \) evaluated for \( x = \hat{x} \).

One approach to detection can be based on the fact that if the model is correct, \( \hat{J} \) is a random variable whose probability distribution can be calculated (at least approximately) if the measurement errors \( \alpha \) are assumed to be Gaussian and (small).

Thus if a value of \( \hat{J} \) is obtained which lies on the tails of this probability distribution, it can be assumed that the model is no good, i.e., an error in \( \tilde{Y} \) or \( \theta \) has been detected.

A different approach to detection does not rely on probability distributions but requires more calculation. To illustrate the method, consider the case of detecting a lost line. Assume that there are \( K \) transmission lines in the static-structure model but that one line may actually be out of service. Let \( \hat{Y}_0 \) denote the admittance matrix with all \( K \) lines in the model, and let \( \hat{Y}_k \) denote the admittance matrices with the \( k \)th line removed from the model, \( k = 1 \cdots K \). Thus there are \( K + 1 \) possible models for the observations given by

\[
z = f_k(x) + \alpha, \quad k = 0 \cdots K
\]

(17)

where \( f_k(x) \) depends on \( \hat{Y}_k, k = 0 \cdots K \). There are also \( K + 1 \) possible state estimates \( \hat{x}_k, k = 0 \cdots K \) depending on which model is used, where \( \hat{x}_k \) is the value of \( \hat{x} \) which minimizes \( J_k(x) \) which is \( J(x) \) for \( f(x) = f_k(x) \).

Let \( \hat{J}_k \) denote \( f_k(x) \) evaluated for \( x = \hat{x}_k \). \( \hat{J}_k \) can be viewed as a measure of how good a fit the static structure model with \( \hat{Y}_k \) (and \( f_k(x) \)) provides to the observation \( z \). If it turned out that for some \( k \), \( J_k << J_0 \), then it would be reasonable to assume that the observations \( z \) were made on a network with line \( k \) removed. Hence a modeling error caused by a lost line (whose loss is not modeled) can be detected by evaluating \( J_k, k = 0 \cdots K \). This concept can be extended to the problem of detecting bad data points, i.e., errors in the modeling of \( \theta \). To illustrate, assume \( \theta \) is diagonal. Then a particular measurement can be modeled as a bad data point by making the corresponding main diagonal of \( \theta \) equal to zero. This leads to a set of possible models and corresponding \( \hat{J}_k, k = 1 \cdots \) which can be used as before.

The detection logic is presented in terms of reasonability arguments. However, they are actually drawn from the theory of hypothesis testing and can be given a more formal mathematical base.

**Identification**

After a modeling error has been detected, the next step is to identify the error so that it can be corrected. The detection logic of (17) also identifies the lost lines. The problem of identifying modeling errors caused by erroneous transmission line admittances requires an extension of the concept, wherein \( \hat{J} \) is viewed as a function of the line admittances and identification is accomplished by finding the admittance values which minimize \( \hat{J} \).
The identification problem can be viewed as a state-estimation problem, where the state vector has been expanded to include unknown model parameters as well as the bus voltage magnitudes and angles. The special term, identification, is used to reduce confusion.

Discussion of Theory

Return first to the simple example of Fig. 1. $x$ is the weighted sum of the anmeter and voltmeter readings, $z_1$ and $z_2$. The weights are determined by the error variances $\theta_{kk}$, the resistance values $R_k$, and the network structure. If $R_1 = 1$, and if $\theta_1 > \theta_2$ (meter 1 noisier than meter 2), then the more accurate meter reading $z_1$ is given the most weight, i.e., has most effect on the estimate. However, if $1 > R_0$, the more inaccurate measurement $z_1$ may be actually given more weight. Thus as far as the estimate is concerned, the weighting due to the $\theta_{kk}$ may be overcome by geometry and network effects. This weighting effect carries over to the general case. Thus the state estimator algorithm (15) is a way of combining measurements of many types into a single estimate in such a way that the importance of any one measurement is automatically determined. The need to combine both pseudo and actual real-time measurements of many types makes this automatic weighting very important.

The iteration sequence (15) depends on the matrix $\Sigma$ which exists only if the matrix $F'\theta^{-1}F$ has an inverse. Let $K_1$ denote the dimension of the observation $z$, and let $K_2$ denote the dimension of the state $x$. $\Sigma$ does not exist if $K_1 > K_2$. If $K_1 = K_2$ and if $F^{-1}$ exists, (15) becomes

$$x_{n+1} = x_n + F^{-1}(z - f(x_n)). \tag{18}$$

If $z$ consists of specified generator and load bus watts, vars, and voltages, this iteration is approximately the Newton-Raphson method which has been successfully used to obtain conventional load flows [4]. If $K_2 > K_1$, some of the measurements are redundant. Thus an alternate to using (15) is to throw away redundant observations until $K_2 = K_1$, and then do a conventional load-flow calculation as typified by (18). However, this alternate approach is not recommended for the following reasons. Throwing away of information reduces the overall accuracy. It is difficult to decide which measurements to ignore; for example, is a watt meter measurement on an EHV transmission line more or less valuable than knowledge of the nominal voltage level at a load point? Equation (18) is not much easier than (15) to implement on a computer. Finally and perhaps most important, the redundant measurements are the quantities which enable detection and identification.

The importance of the use of redundancy for detection and identification should be emphasized. Data processing algorithms for handling large quantities of telemetered data made on a complex changing system must have internal checks to detect modeling errors such as unexpected system changes and bad data points. Without such tests, very little faith can be given to the estimate.

The interpretation of the matrix $\Sigma$ as the covariance matrix of the errors in $x$ has important applications. The value of any estimate is greatly enhanced if its accuracy is known. $\Sigma$ is also very useful in initial design and development as $\Sigma$ can be calculated before the state estimator has been implemented. $\Sigma$ can be used to study where meters should be placed and what type and accuracy of information is actually needed from IS. $\Sigma$ is not a perfect measure of performance, but it is a very useful design tool.

Initial Tests

The behavior of the static-state estimator has been investigated by digital-computer simulation. The nature and results of these initial tests are briefly summarized. Many numerical results and much more extensive discussions can be found in [5].

The computer program can be viewed as a Monte Carlo simulation. It can be divided into four blocks.

1) Given the desired network and bus powers, establish the true bus voltages by a conventional load flow.

2) Given the desired meter placement, types, and accuracies, simulate a set of meter readings by first calculating the readings perfect meters would make and then adding errors obtained from a random number generator.

3) Given the simulated measurements; calculate the best estimate of the voltages at the buses using the static-state estimator.

4) Analyze the errors by subtracting the true bus voltages from the estimated bus voltages and repeating blocks 2) and 3) many times using different random errors so that an average squared error can be obtained.

The initial tests to be discussed here were based on a five-bus system which is an example of [4, p. 284]. Many trials were made with various numbers, types, locations, and accuracies for the meters which could measure real power, reactive power, or voltage at any bus. A measurement with a large error variance was assumed to be a pseudomeasurement.

Performance depended heavily on meter placement, accuracy, and type. For a good choice of meter placement and types, the iteration of (15) converged rapidly with an average rate approximately independent of initial starting value or variance of the measurement error. However, the actual convergence behavior could depend on the particular noise values. For a bad choice of meter placement and type, convergence was sometimes slow or nonexistent. Meter placement and type as well as meter accuracy affected the accuracy of the final estimate. The effect of adding or removing one meter was sometimes dramatic and sometimes unobservable; it all depended on the situation. Usually variometers were more effective than voltimeters but a critically located voltmeter could be a big help in some cases. Pseudomeasurements with large error variances could have major or minor effects, once again depending on location and type.

At no time in the initial tests did the iterations appear to converge to a local minimum or hang up on a flat spot. However, there is of course no way to be positive this did not happen without evaluating $J(x)$ for all possible values of $x$.

A wide range of meter placement and types was explored in the hope of developing some rules of thumb for choosing meter placement and types which are economical and yet provide good estimates. Unfortunately, no such insight that could be extrapolated to larger systems was obtained. This is the main reason a larger than five-bus system was not investigated. We are convinced the static-state estimator will work well on any order system provided that good meter placement and types are chosen. However, at our present level of insight, we would have to study the exact system to see if a particular meter placement pattern is good. (Of course, if cost is no consideration, complete direct metering of all buses and lines in OS and IS is good.)

The simulations did verify the fact that $\Sigma$ does indeed approximate the average squared error if the errors are small. In fact, most of the studies on meter placement and accuracy were based on calculating only $\Sigma$ rather than using the program in a complete Monte Carlo mode of operation.
Conclusions

The basic concepts underlying a static-state estimator and the corresponding detection-identification logics have been presented. The paper can be viewed as an exercise in modeling and formulating the power system problem so that classical estimation and detection theory can be employed. The resulting equations are reasonable and worked well during simulation.

Many important aspects of the problem were not discussed. Some of these will be considered in [2] and [3].

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Power System Static-State Estimation,
Part II: Approximate Model

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Abstract—The static state of an electric power system is defined as the vector of the voltage magnitudes and angles at all network buses. The static-state estimator is a data-processing algorithm for converting redundant meter readings and other available information into an estimate of the static-state vector. Discussions center on an approximate mathematical model (related to the dc load-flow model). This model yields noniterative-state estimation equations, simplified prediction of effects of network and generation-load pattern changes on network flow, and simplified detection and identification of modeling errors. Results of some initial computer studies on the real-power-voltage angle portion of the approximate model are discussed.

Introduction

This paper is the second of a three-part sequence on a static-state estimator and related detection and identification problems. The static-state estimator is a data processing algorithm for use by a digital computer to reduce meter measurements and other information on an electric power system into an estimate of the system's steady-state or static-state vector. The static state is a vector composed of the voltage magnitudes and angles at all the buses.

In the first paper [3] mathematical models and general state estimation, detection, and identification algorithms were discussed. In this paper, an approximate mathematical model and the resulting simplifications in estimation, detection, and identification are discussed. In the following paper [4] various implementation problems associated with dimensionality, computer speed and storage, and the time-varying nature of actual power systems will be discussed.

The following notational conventions are used. A bold face letter denotes a vector or matrix. All vectors are column vectors. A prime denotes matrix transpose, and $-1$ denotes matrix inverse. The letter $E$ denotes the expectation or averaging operation on a random variable or vector.

Approximate Model

The mathematical model developed in [3] is of the form

$$z = f(x) + n$$

(1)

where $x$ is the state vector (voltage magnitude and angle at all buses), $f(x)$ is the nonlinear function of $x$ determined by network admittance matrix $Y$ which gives the measurements of ideal meters, $n$ is the measurement (meter) error vector (zero mean, random vector with error covariance matrix $Q$), and $z$ is the vector of measurements fed into computer. Certain approximations of $f(x)$ yield an approximate mathematical model which is closely related to the dc load-flow concept.

Assume there are $N_b$ buses and $N_t$ transmission lines. Let $P_k$ and $Q_k$ denote the real and reactive electric power into $k$th bus, $k = 1 \cdots N_b$ from load or generator sources. Let $V_k$ and $\delta_k$

References