



MEDNARODNA PODIPLOMSKA ŠOLA JOŽEFA STEFANA  
JOŽEF STEFAN INTERNATIONAL POSTGRADUATE SCHOOL

# Social Network Analysis

Course: New Media and Knowledge Management

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# Overview

1. Introduction to social network analysis
2. Domain: Co-authorship graph:  
ILPnet2
3. Cohesion
4. Brokerage
5. Ranking



# SOCIAL NETWORK ANALYSIS

- Social network analysis focuses on interpreting patterns of social ties among people, groups of people, organizations, and countries.
- A typical domain is a group of individuals and their characteristics and the structure of their ties.
- Goals:
  1. COHESION
    1. How many separate research groups do exits?
    2. How connected are the researchers with each other?
  2. BROKERAGE
    1. Who are the most influential authors?
    2. Who are the authors who are responsible for ideas trading across the community?
  3. RANKING
    1. Who are the most prestigious authors in the area?
    2. Is there a census on who are the most important authors

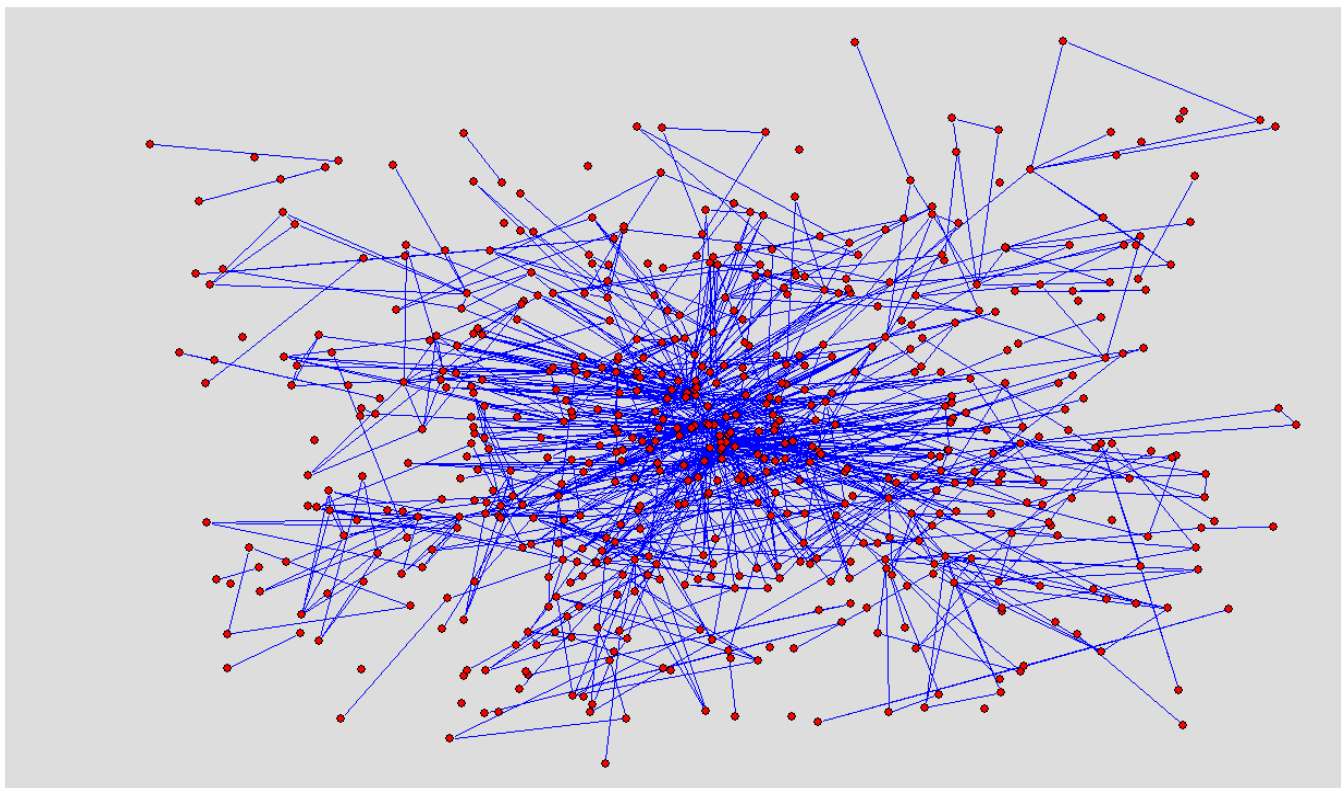


# THE DOMAIN: ILPnet2

- Network of Excellence in Inductive Logic Programming (1998-2002)
- <http://www.cs.bris.ac.uk/~ILPnet2/>
- Basic characteristics: 589 authors, 1046 co-authorships, 1147 publications from 1970 to 2003



# ILPnet2 network



[illegible]



# COHESION

What is Cohesion?

1. Density
2. Degree
3. Components
4. Cores



# COHESION

- COHESION = an attractive “force” between individuals
- SOCIAL NETWORKS  $\Rightarrow$  dense pockets of people who »stick together« = COHESIVE SUBGROUPS.
- The first concern of social network analysis  $\Rightarrow$  to investigate who is related and who is not.



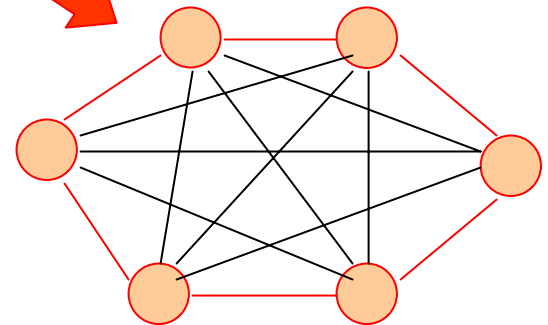
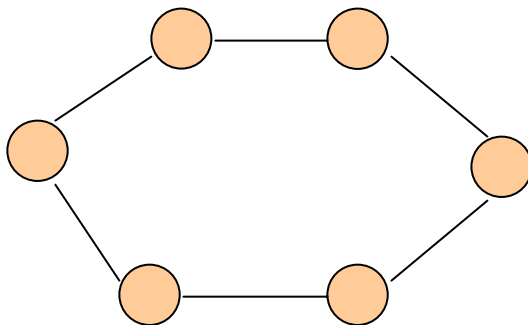
# DENSITY

Density of the network = the number of lines in a simple network, expressed as a proportion of the maximum possible number of lines

- all possible lines = 15

- number of lines = 6

- **Density =  $6/15 = 0.4$**





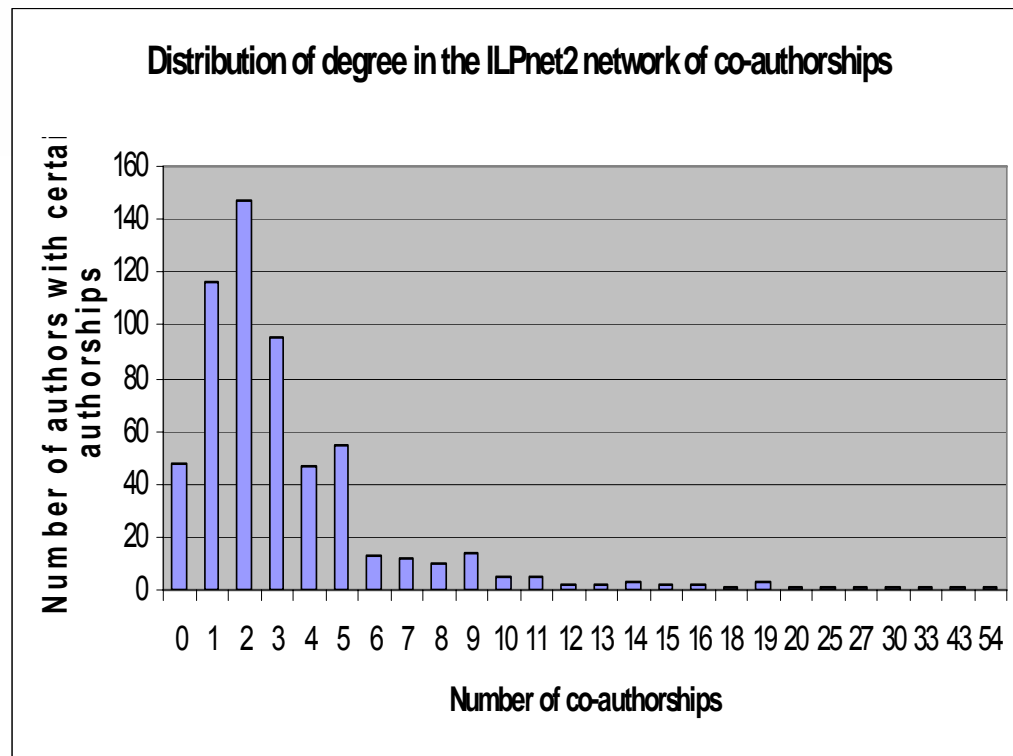
# DENSITY

- inversely related to network size  $\Rightarrow$  the larger the social network, the lower the density
- ILPnet2 network Density = number of lines / maximum possible number of lines =  
$$= 1046 / 173166 = 0.0060$$



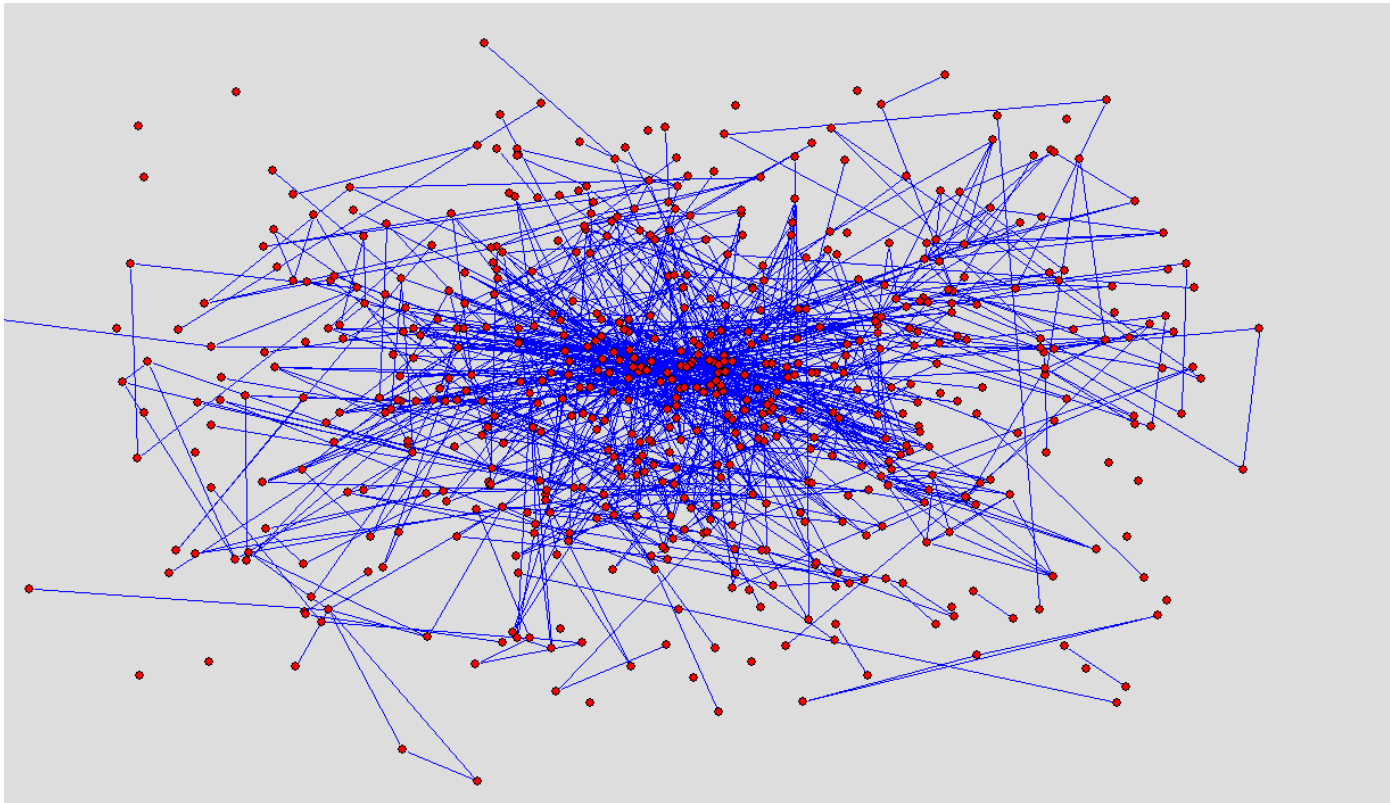
# DEGREE

- A *degree* of a vertex = the number of lines incident with it.



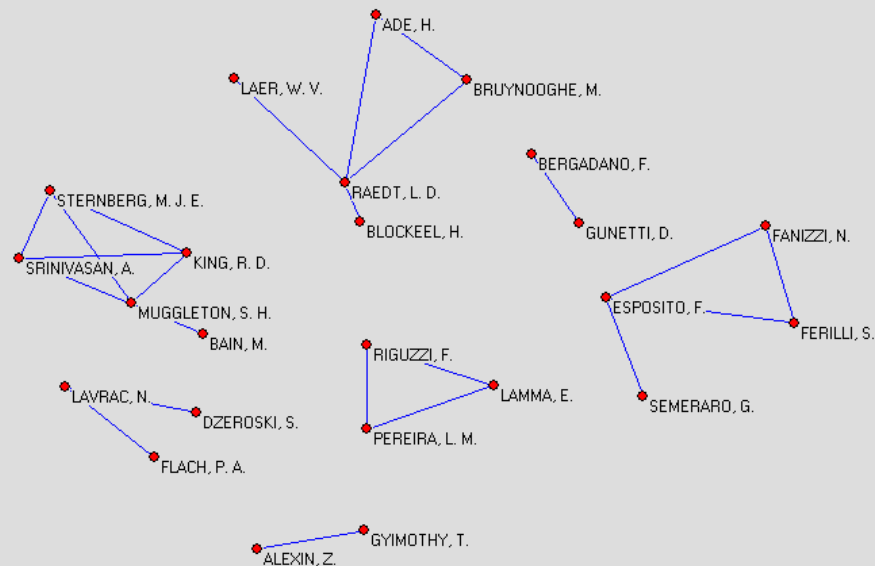


# ILPnet2 network



# ILPnet2 network

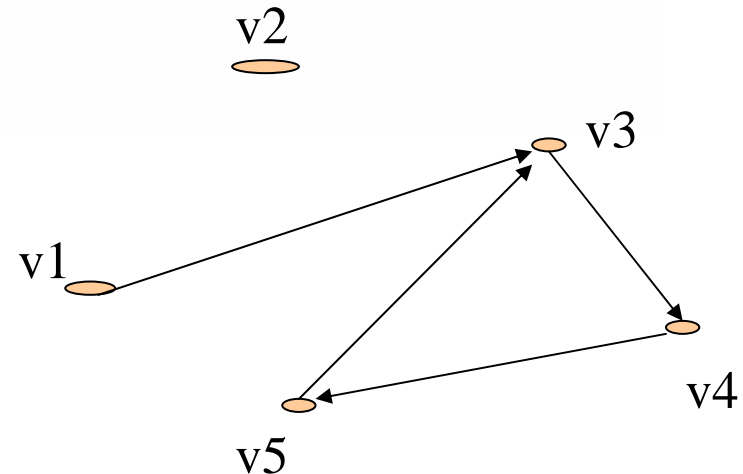
- removed lines with value lower than 10 and reduced for vertices with degree lower than 1





# WALKS AND PATHS

- semiwalk = we don't consider the direction of the arcs (from  $v5 \rightarrow v3 \rightarrow v1$ )
- walk = we have to follow the directions of the arcs ( $v5 \rightarrow v3$ )
- semipath = semiwalk in which no vertex in between the first and last vertex of the semiwalk occurs more than once ( $v5 \rightarrow v3 \rightarrow v4 \rightarrow v5 \rightarrow v3$ )
- path = walk in which no vertex in between the first and last vertex of the walk occurs more than once ( $v5 \rightarrow v3$ )



*ILPnet2 network is undirected*



*strongly/weakly connected  
network*



# COMPONENTS

- Components identify cohesive subgroups in a straightforward manner - each vertex belongs to exactly one component.
- weakly connected networks = all vertices are connected by a semipath
- strongly connected networks = all vertices are connected by a path

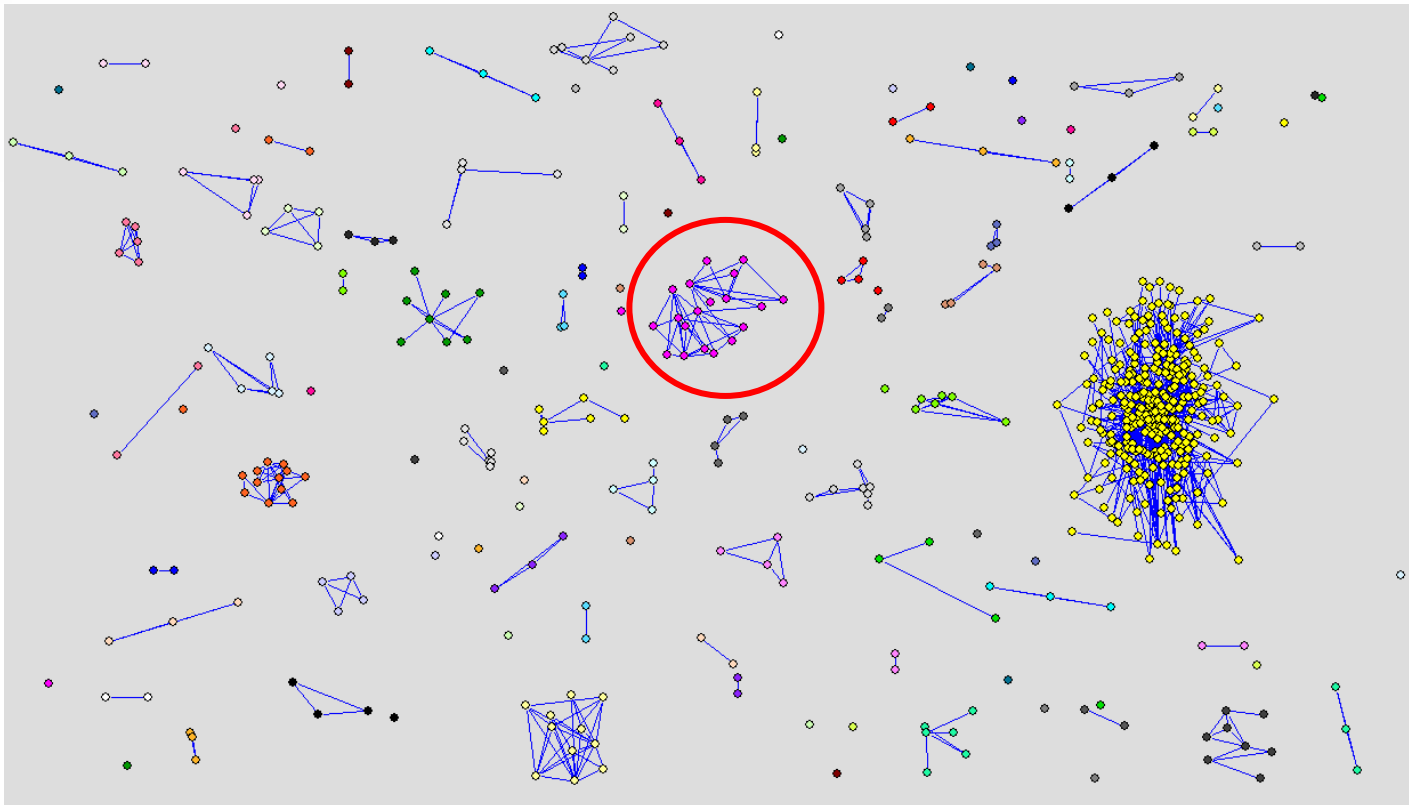


# COMPONENTS

- A weak component is a maximal weakly connected subnetwork
- A strong component is a maximal strongly connected subnetwork.

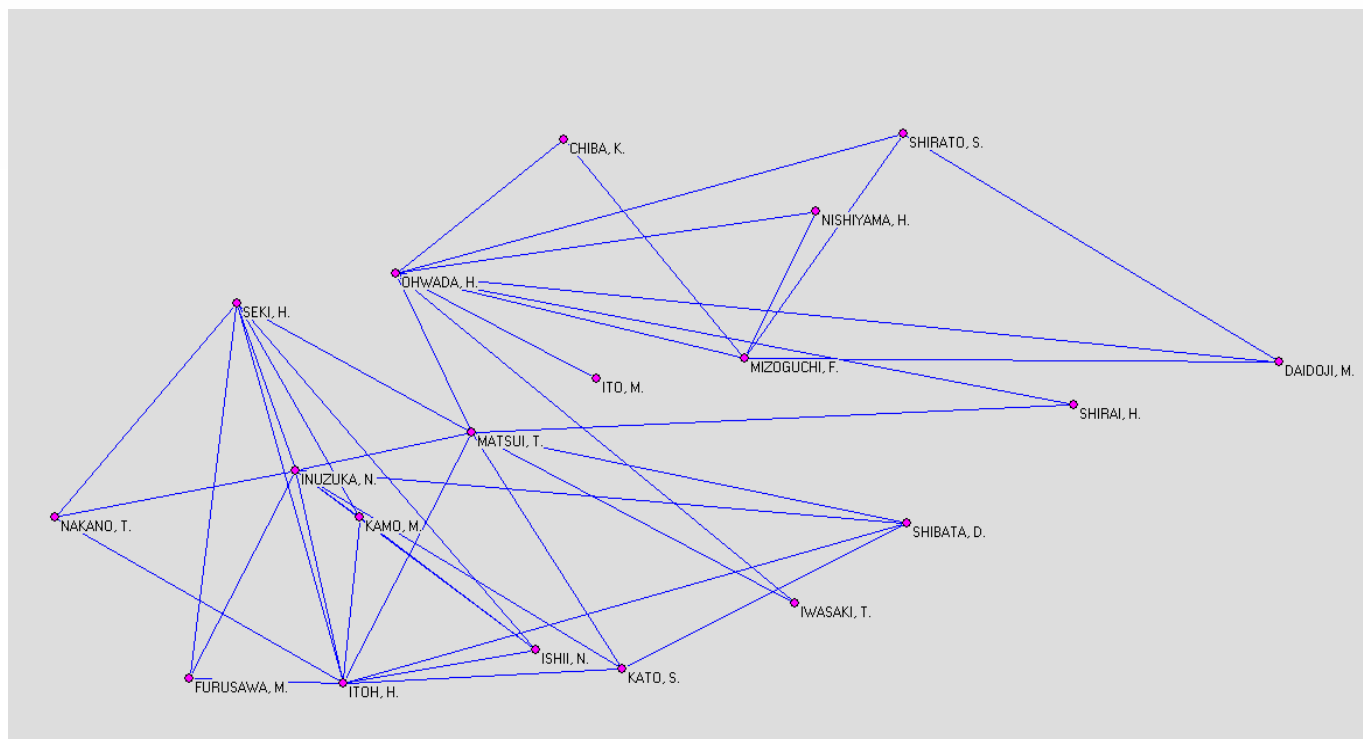


## 110 Components in ILPnet2 network

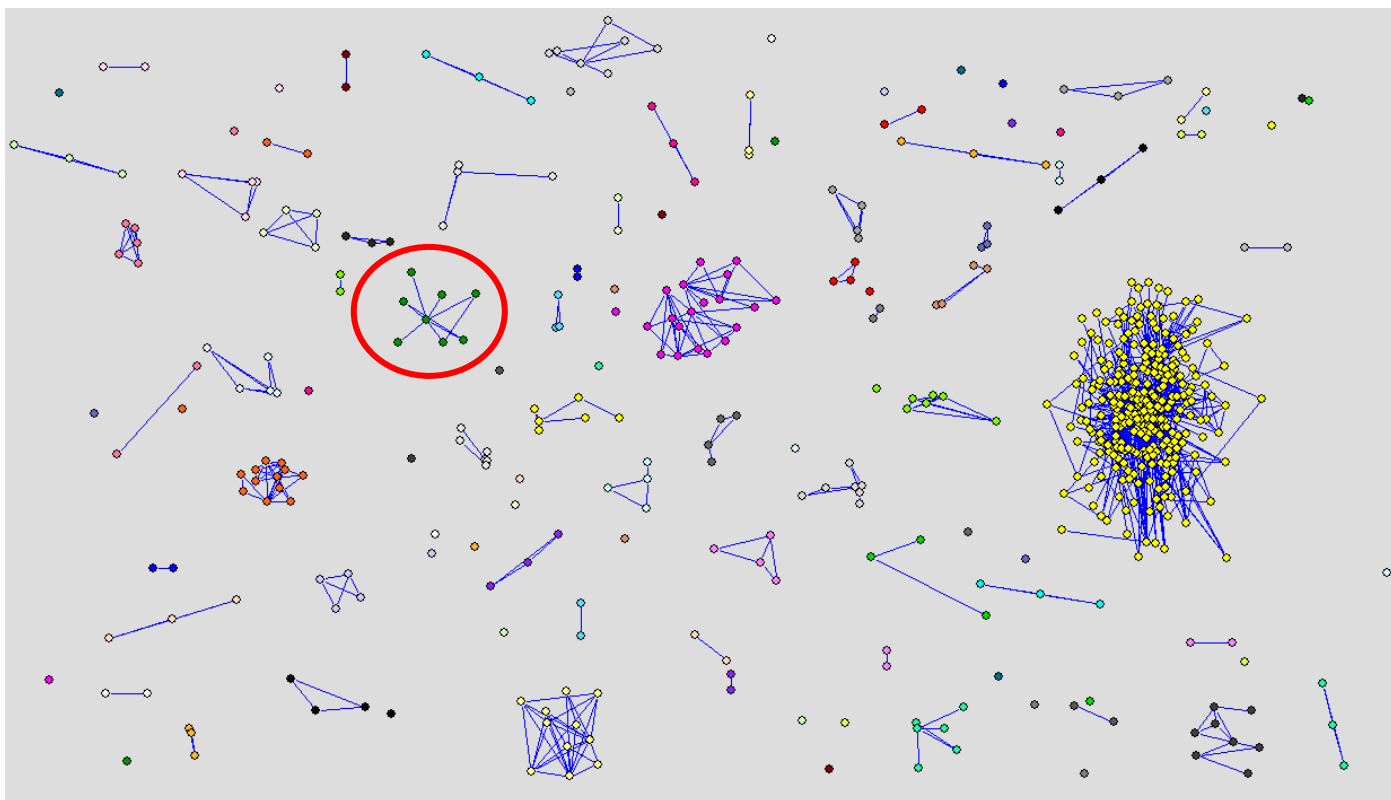




# Zoomed component

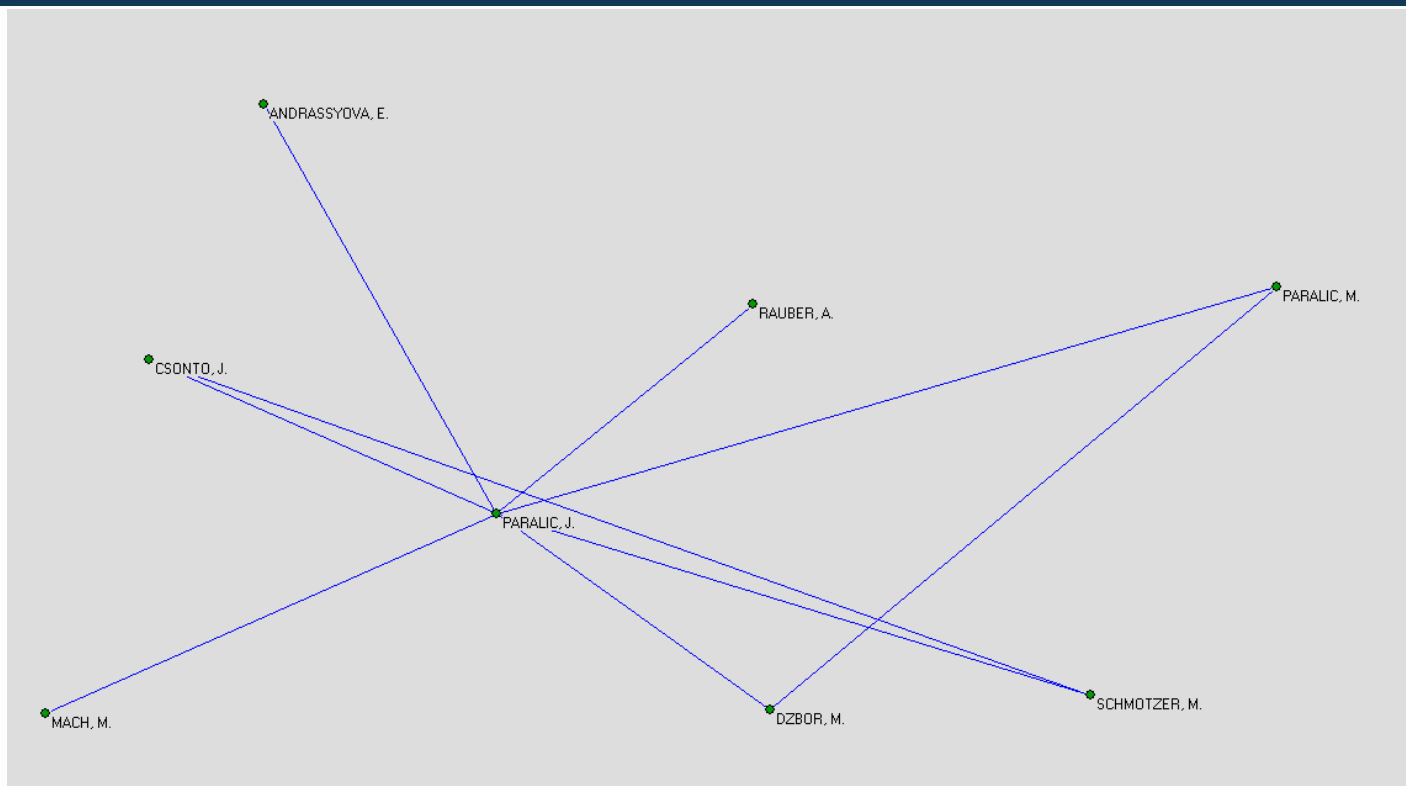


## 110 Components in ILPnet2 network

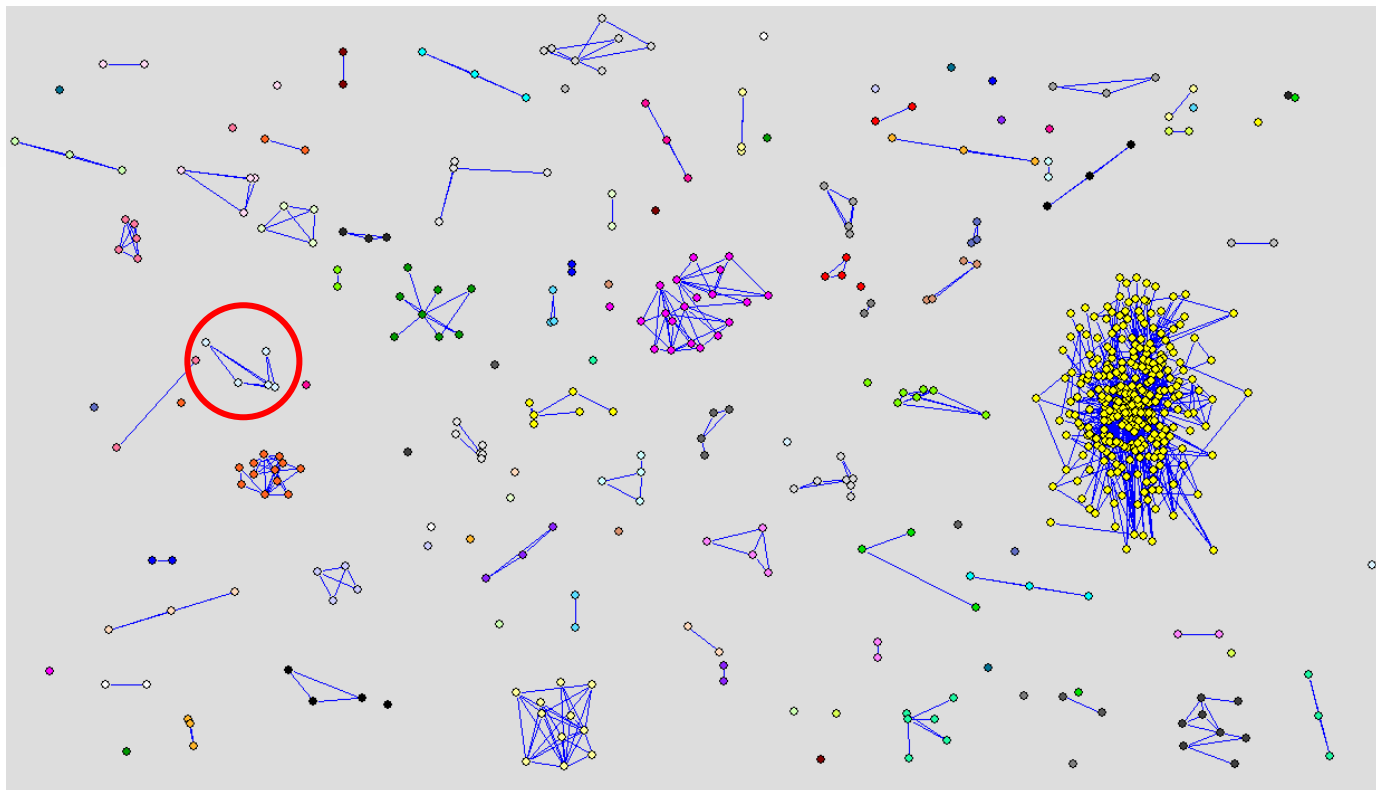




# Zoomed component

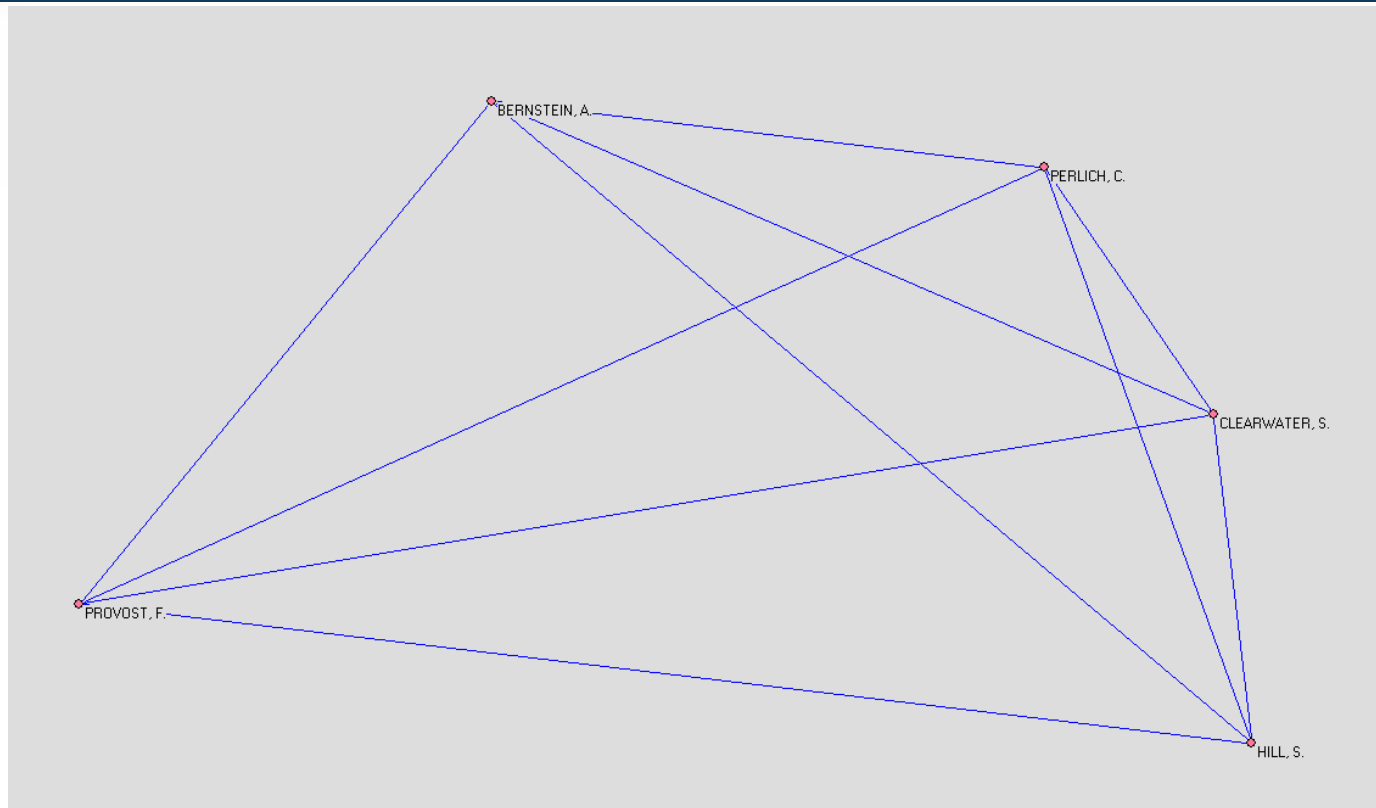


## 110 Components in ILPnet2 network

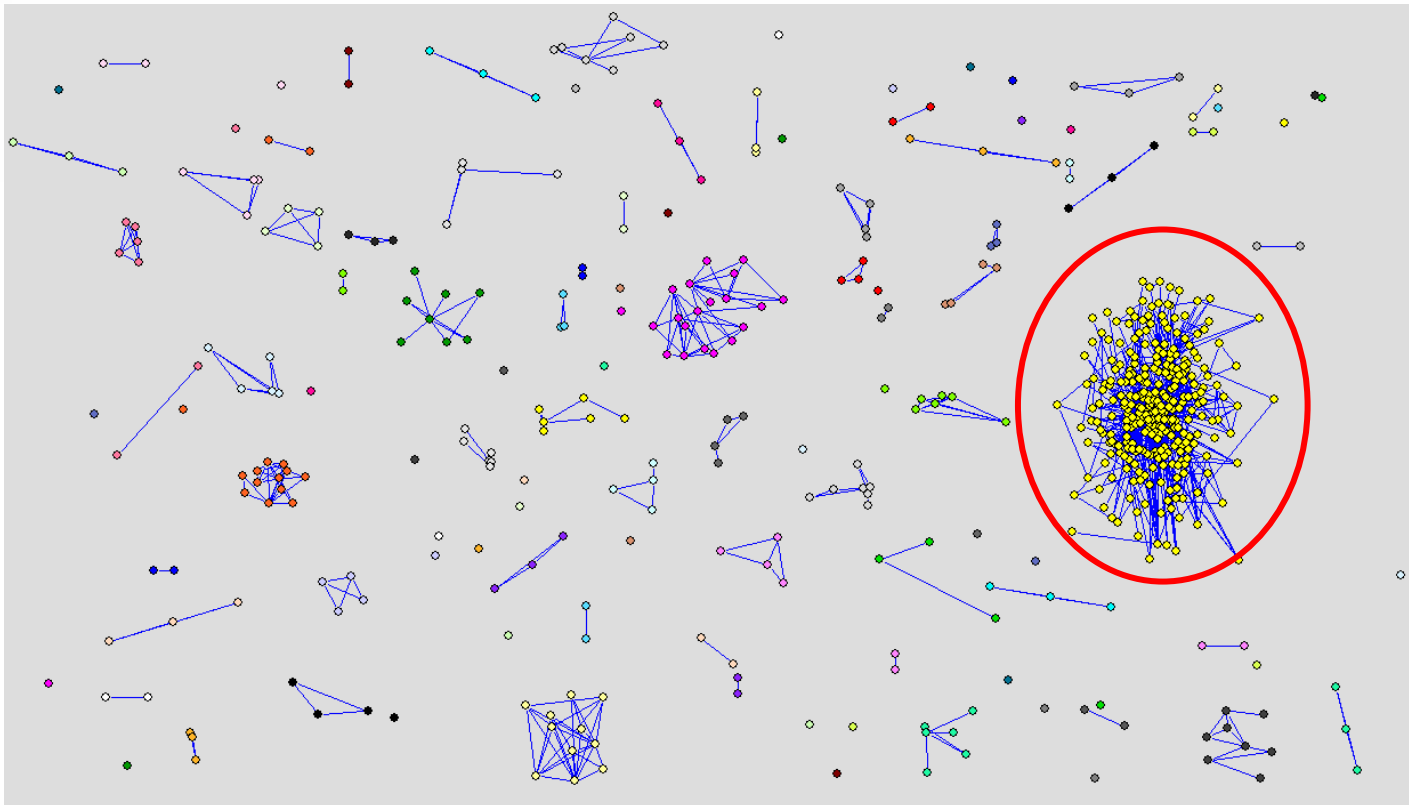




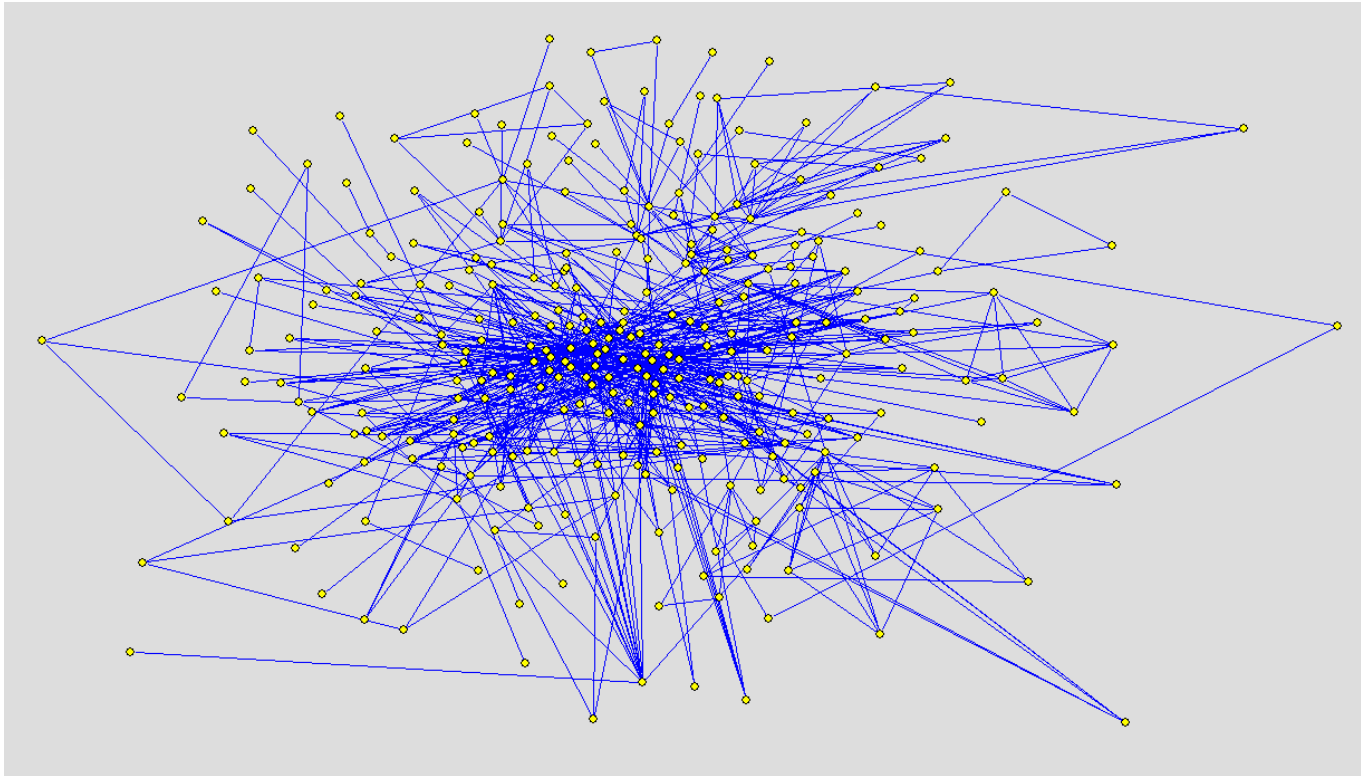
# Zoomed component



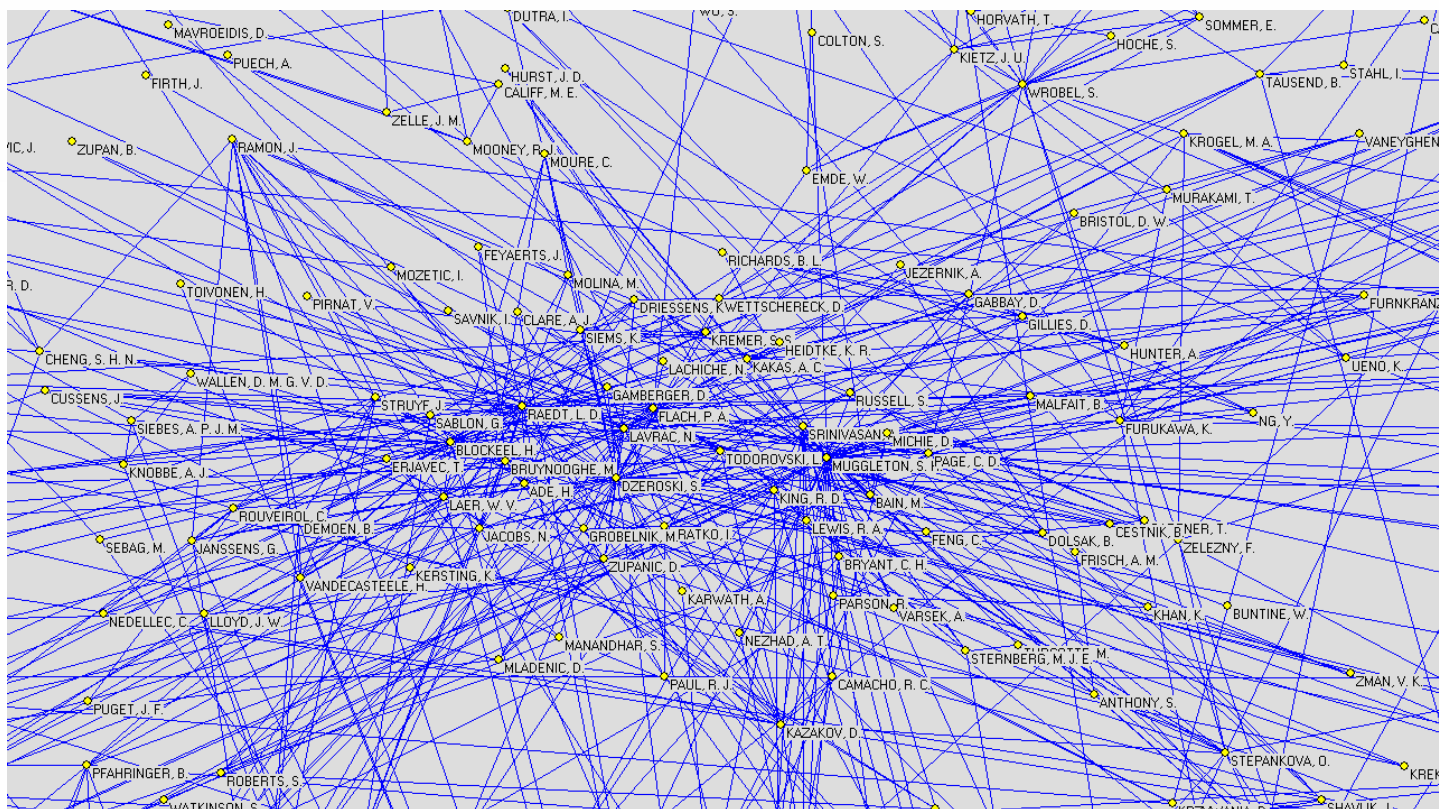
## 110 Components in ILPnet2 network



# Zoomed component







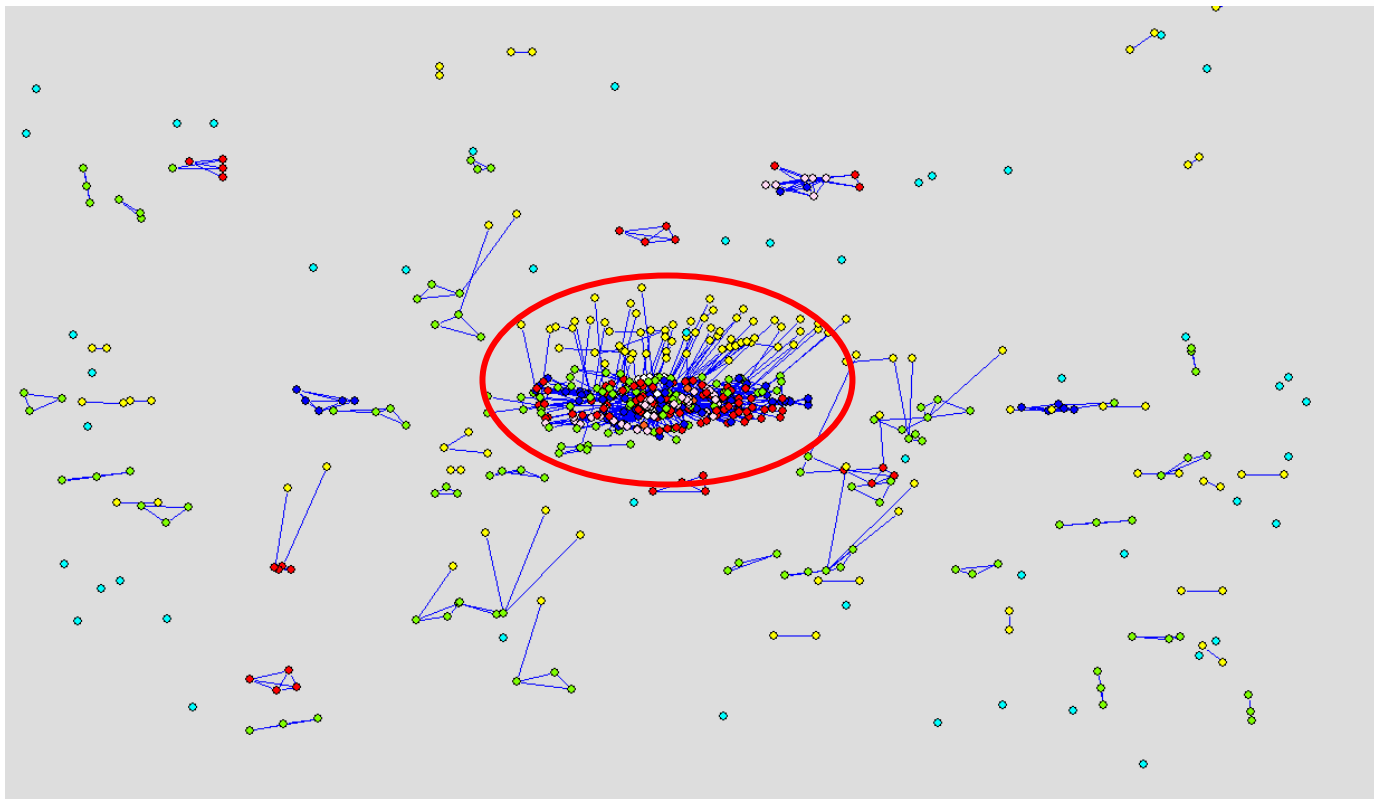


# CORES

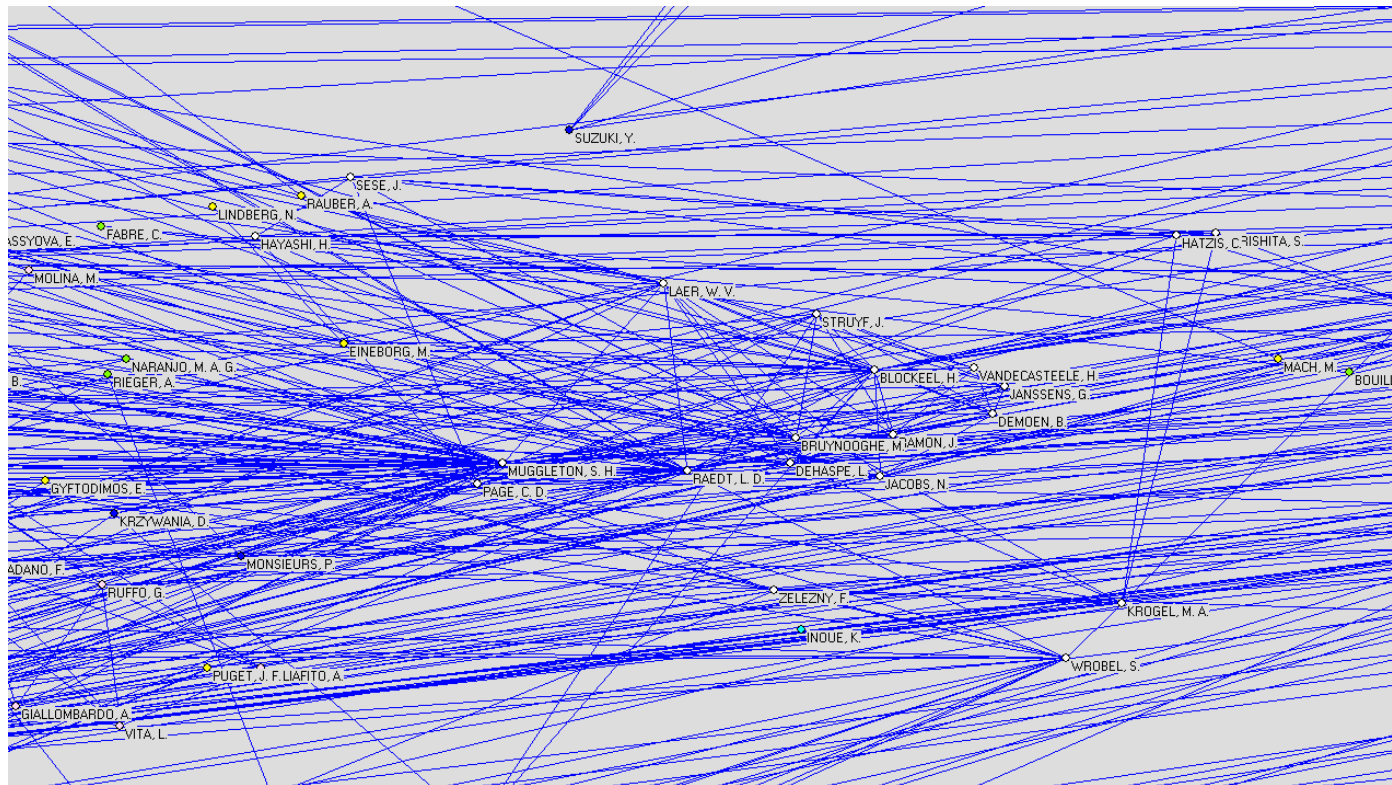
- A  $k$ -core is a maximal subnetwork in which each vertex has at least degree  $k$  within the subnetwork
- A  $k$ -core is not necessarily a cohesive group itself.
- Are the important people all present in the same component or are they scattered around?



## ILPnet2 network with 7 cores – each color represents one core



# Zoomed $k$ -core

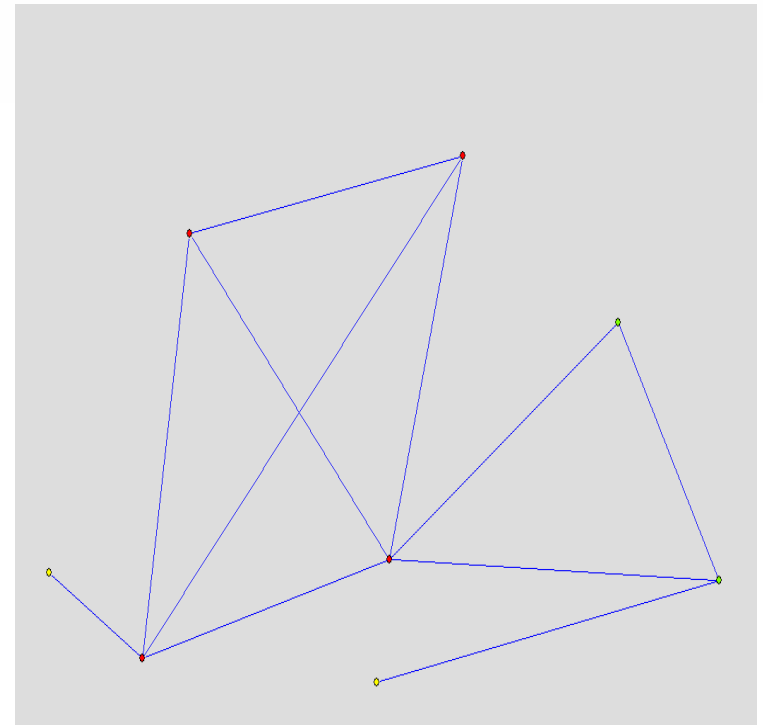




# CORES

## **Nested cores:**

**a vertex in a 3-core (red-colored dots) is also part of a 2-core (green-colored dot), but not all members of a 2-core belong to a 3-core**





# CLIQUEES

- Homework:
  - What is a clique?
  - What is clique hierarchy?
  - What is the interpretation of a clique?



# BROKERAGE

- **Center and Periphery:**

- Nodes in the center have more information and are share the information easier
- Centrality Measures
  - Degree centrality/centralization
  - Closeness centrality/centralization
  - Betweenness centrality/centralization

- **Brokers and Bridges**

- Nodes which bridge structural holes are more important – Bridges
- Brokers are bridges across many structural holes





## **Degree centrality/centralization reachability of a vertex inside a network**

- The star network is the most efficient structure (given the fix number of lines)
- Network is more centralized if the vertices vary more with respect to their centrality. More variation in centrality scores of vertices yields a more centralized network.

### **Defining degree of centralization**

In social network: Who has the more sources of information at its disposal?

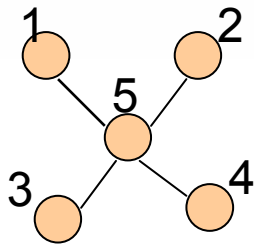
- The degree centrality of vertex is its degree
- Degree centralization of a network is the variation in the degrees of vertices divided by the maximum degree which is possible in the network of the same size





# Degree centrality/centralization

a) **star network** (most efficient) degree centralization:

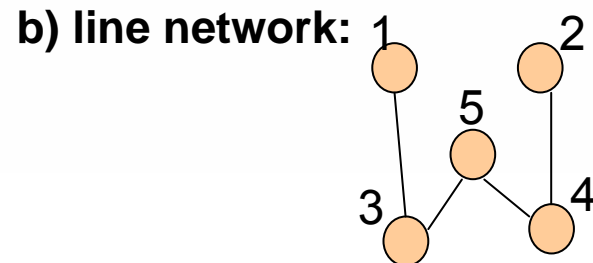


v5 degree = 4 (max degree)

v1 to v4 degree = 1 (min degree)

=> v5 contributes  $1 \times (4-1)$  and v2 to v4 contributes  $4 \times (1-1) \Rightarrow$  so **12 is the maximum degree** variations

=>  $12/12 = 1$  max degree centralization



v1 and v2 degree = 1

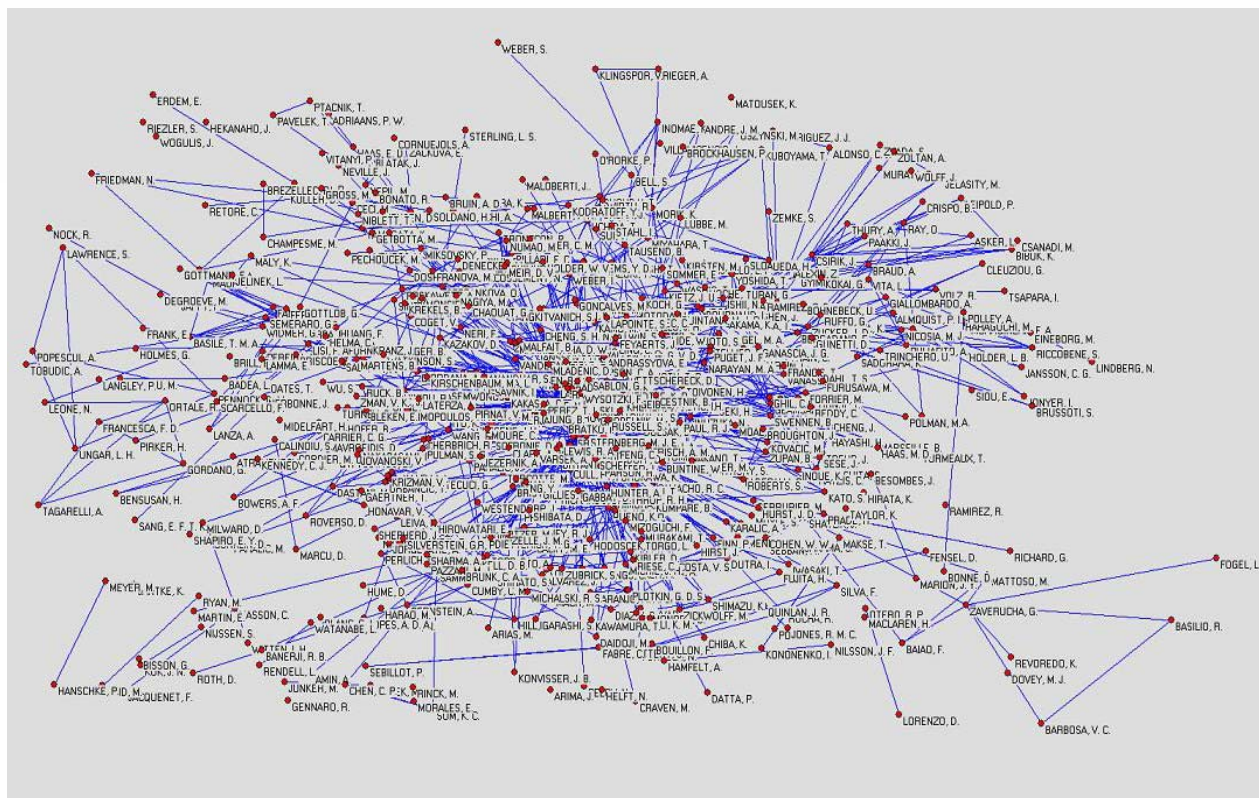
v3, v4 and v5 degree = 2 (max degree in this network)

=> v1 and v2 contributes  $2 \times (2-1)$  and v3 to v5 contributes  $3 \times (2-2)$

=>  $2 / 12$  (max degree in the network of the same size) = 0,17



# Degree centrality/centralization on IIPNet2 (all vertices)





## Degree centrality/centralization on IIPNet2

- **Who are the most central persons in network; who has the most collaborations?**
- First we reduce number of vertices to those connected with min two neighbors
- Net > Transform > Reduction > Degree > All (min. Degree of vertices < 2)
- From 589 to 416 vertices

We remove people who wrote only one article by themselves or pairs of people that wrote one article together



# Degree centrality/centralization on IIPNet2

## Centralization of the network:

Net > Partitions > Degree > All

- All degree centrality of 2. All (recursive) degree reduction of N1 [2] (416)
- -----
- Working...
- -----
- Network All Degree Centralization = 0.10282

## Top 20 central persons in IIPNet2 (sorted using excel)

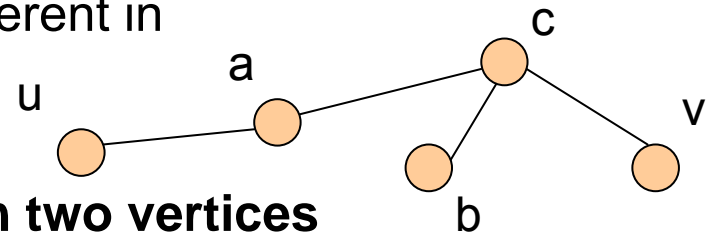
- |                               |                               |
|-------------------------------|-------------------------------|
| 1. 0.1132530 - MUGGLETON,S.   | 11. 0.0409639 - PAGE,C.       |
| 2. 0.1036145 - DZEROSKI,S.    | 12. 0.0385542 - KING,R.       |
| 3. 0.0722892 - BLOCHEEL,H.    | 13. 0.0385542 - JACOBS,N.     |
| 4. 0.0722892 - RAEDT,L.       | 14. 0.0361446 - STEPANKOVA,O. |
| 5. 0.0650602 - LAVRAC,N.      | 15. 0.0337349 - RAMON,J.      |
| 6. 0.0481928 - FLACH,P.       | 16. 0.0337349 - DEHASPE,L.    |
| 7. 0.0457831 - LAER,W.        | 17. 0.0337349 - GYIMOTHY,T.   |
| 8. 0.0457831 - SRINIVASAN,A.  | 18. 0.0337349 - BERGADANO,F.  |
| 9. 0.0433735 - WROBEL,S.      | 19. 0.0313253 - KAZAKOV,D.    |
| 10. 0.0433735 - BRUYNOOGHE,M. | 20. 0.0289157 - ZUPANIC,D.    |



# Distance – Geodesic

**Two vertices (people) are connected if path exists from one to another**

- In undirected network the distance is the number of lines or steps in the shortest path that connect two vertices together
- In directed network distance can be different in reverse way (one-way street example)



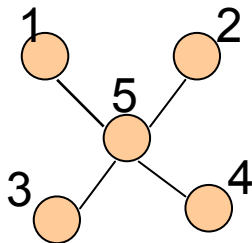
**A geodesic is the shortest path between two vertices**

- The distance from vertex  $u$  to vertex  $v$  is the length of the geodesic  $u$  to  $v$ .



## Distance/Closeness – centrality/centralization

- The closeness centrality of a vertex is the number of all other vertices divided by the sum of all distances between the vertex and all others



$$v5: 4 / 4 = 1$$

$$v1: 4 / 1+2+2+2 = 4 / 7$$

- We see that the problem arises if all vertices are not (strongly) connected!
- In social network: how easy is it for a person to notify the whole network with information.



## Distance – centrality/centralization on IIPNet2

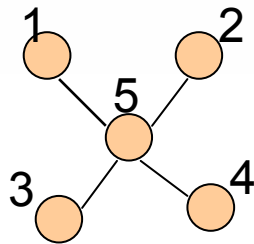
- |                                |                               |
|--------------------------------|-------------------------------|
| 1. 0.3996198 - RAEDT, L.       | 11. 0.3142049 - KAZAKOV, D.   |
| 2. 0.3758981 - DZEROSKI, S.    | 12. 0.3106478 - DEHASPE, L.   |
| 3. 0.3741894 - MUGGLETON, S.   | 13. 0.3071704 - FLACH, P.     |
| 4. 0.3415837 - LAER, W.        | 14. 0.3037700 - RAMON, J.     |
| 5. 0.3360069 - PAGE, C.        | 15. 0.3015446 - CUSSENS, J.   |
| 6. 0.3332861 - JACOBS, N.      | 16. 0.2993516 - BRATKO, I.    |
| 7. 0.3279748 - LAVRAC, N.      | 17. 0.2961211 - DRIESSENS, K. |
| 8. 0.3215691 - WROBEL, S.      | 18. 0.2919208 - WEBER, I.     |
| 9. 0.3178443 - BLOCKEEL, H.    | 19. 0.2898651 - MOURE, C.     |
| 10. 0.3142049 - BRUYNOOGHE, M. | 20. 0.2898651 - MOLINA, M.    |





## Betweenness centrality/centralization

- The betweenness centrality of a vertex is the proportion of all geodesics between pairs of other vertices that include this vertex



v5: 1

v1 to v4:0

- In social network : to what extent may a person (vertice) control the flow of informaton due to the his / her position inside the communication network?





## **Betweenness centrality/centralization on IIPNet2**

**Q: I discovered something new in the area, how likely is that the information spreading will get blocked and not the whole network will be informed**

- Net > Vector > Centrality > Betweenness
- Network Betweenness Centralization = 0.09198

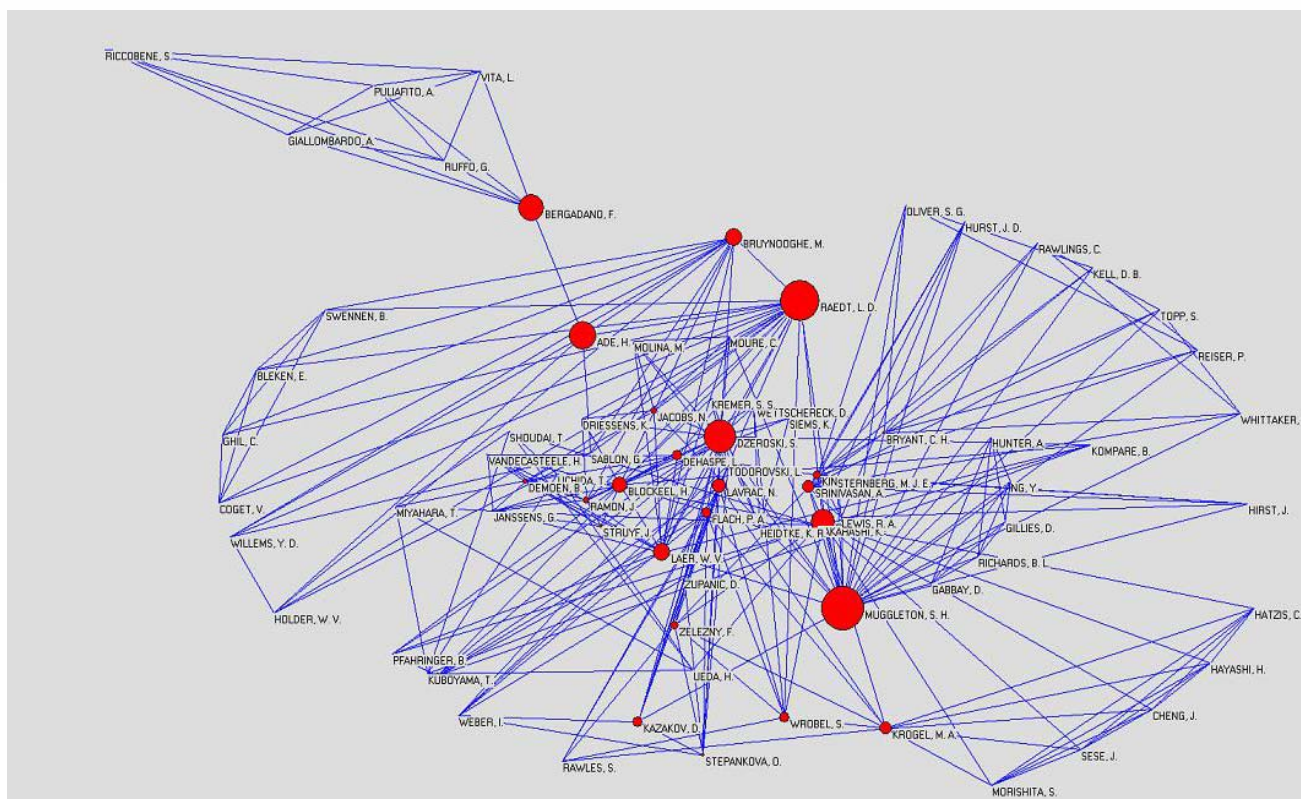


## Betweenness centrality/centralization on IIPNet2

- This are top twenty persons with ability to control information spreading

1. 0.0931813 - MUGGLETON, S.
2. 0.0742590 - RAEDT, L.
3. 0.0546139 - DZEROSKI, S.
4. 0.0459601 - WROBEL, S.
5. 0.0375969 - PAGE, C.
6. 0.0343720 - FLACH, P.
7. 0.0234424 - ADE, H.
8. 0.0218422 - BLOCKEEL, H.
9. 0.0192772 - LAVRAC, N.
10. 0.0181330 - STEPANKOVA, O.

11. 0.0178831 - ROUVEIROL, C.
12. 0.0170300 - BERGADANO, F.
13. 0.0157335 - BOSTROM, H.
14. 0.0153786 - FURUKAWA, K.
15. 0.0152876 - BAIN, M.
16. 0.0143174 - GYIMOTHY, T.
17. 0.0119656 - SHAVLIK, J.
18. 0.0110439 - CHENG, S.
19. 0.0107273 - SRINIVASAN, A.
20. 0.0106385 - LAER, W.



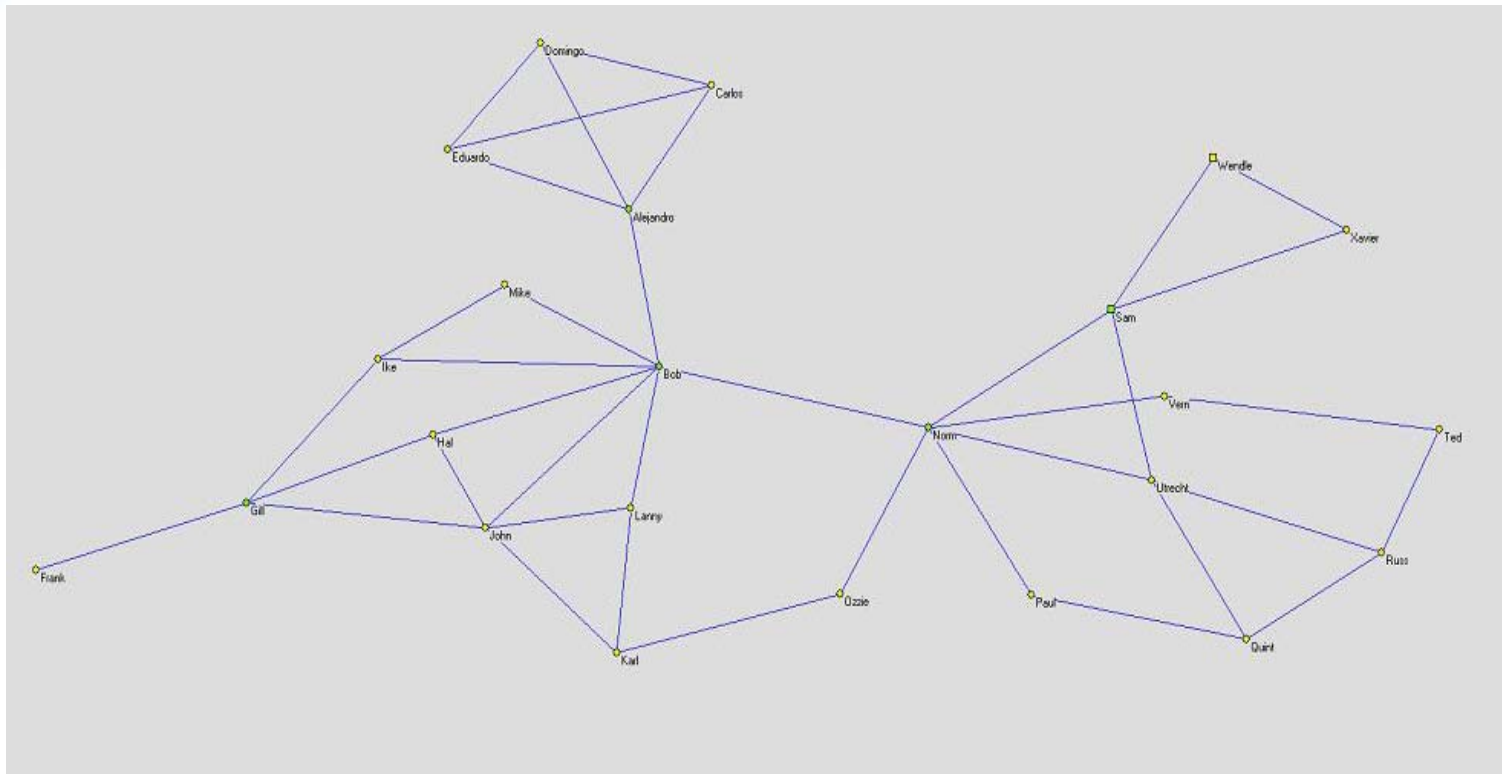


# Bridges

- The bridges and lines who bridge structural holes between other have more control and perform better
- A bridge is a line whose removal increases the number of components in the network
- Deleting a vertex from a network means that the vertex and all lines incident with this vertex are removed from the network
- A cut-vertex is a vertex whose deletion increases the number of components in the network
- A bi-component is a component of minimum size of three that does not contain a cut-vertex

# Bridges

## simple example



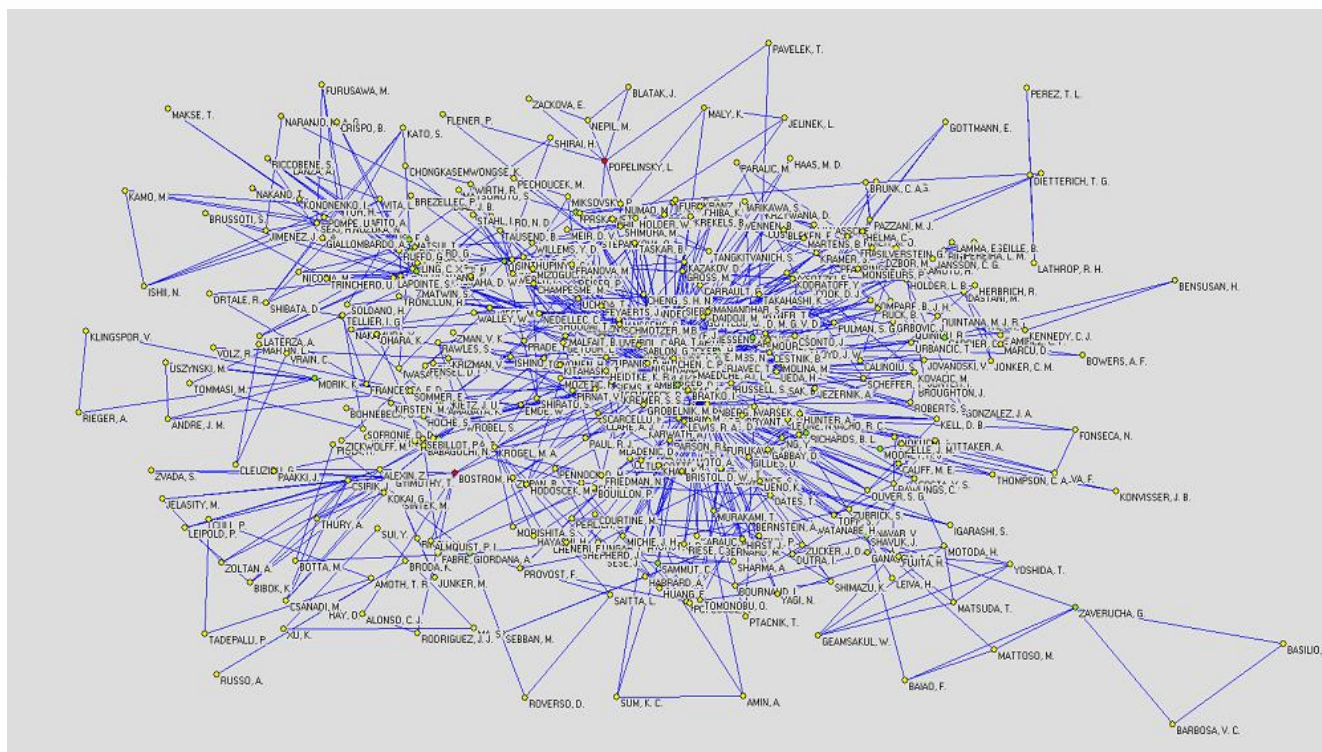


## Bridges on IlpNet2

- Who are the bridges and lines in IlpNet2 who bridge structural holes
- $\text{Net} > \text{Components} > \text{Bi-Components}$  (with a minimum size of 2 so we can look for lines that represents bridges)

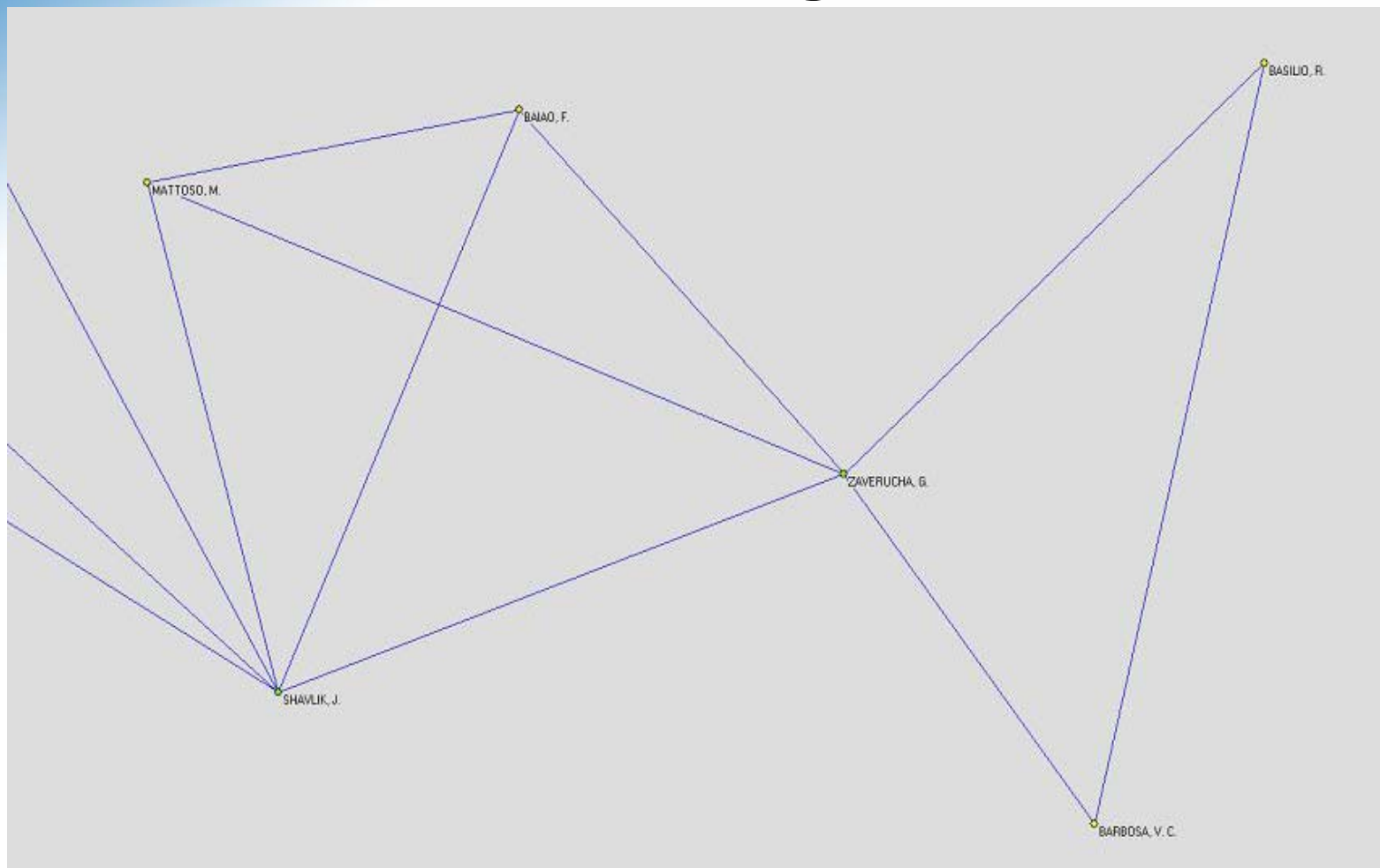


# Bridges on IIPNet2





# Bridges







# Bridges IIPNet2

- **Who are the bridges and lines in IIPNet2 who bridge structural holes, articles that two persons work together on?**
- **Net > Components > Bi-Components (with a minimum size of 2 so we can look for lines that represents bridges)**

• Root (449)	1. 11 - MUGGLETON, S.	14. 3 - KIETZ,	J.
• 1 (3)	2. 7 - FLACH, P.	15. 3 - SEBAG,	M.
• 2 (3)	3. 6 - BOSTROM, H.	16. 3 - KRAMER, S.	
• 3 (11)	4. 6 - CHENG, S.	17. 3 - FURUKAWA, K.	
• 4 (4)	5. 5 - ROUVEIROL, C.	18. 3 - KAKAS,	A.
• 5 (3)	6. 5 - PARALIC, J.	19. 3 - GIORDANA, A.	
• 6 (4)	7. 4 - ZAVERUCHA, G.	20. 3 - MORIK,	K.
• ...	8. 4 - VRAIN, C.	21. 3 - PAZZANI, M.	
• 25(2)	9. 4 - RAEDT, L.	22. 2 - RIGUZZI, F.	
• 26(2)	10. 4 - PAGE, C.	23. 2 - WROBEL, S.	
	11. 4 - SAMMUT, C.	24. 2 - HORVATH, T.	
	12. 3 - POPELINSKY, L	25. 2 - TURAN,	G.
	13. 3 - OHWADA, H.		

- Bridges are bi components of size two in an undirected network, so we can easily find them



# RANKING

- I. Prestige
  - Structural prestige, social prestige, correlation
  - Ways of calculating structural prestige
- II. Ranking
  - Triad census
  - Acyclic decomposition
  - Symmetric-acyclic decomposition

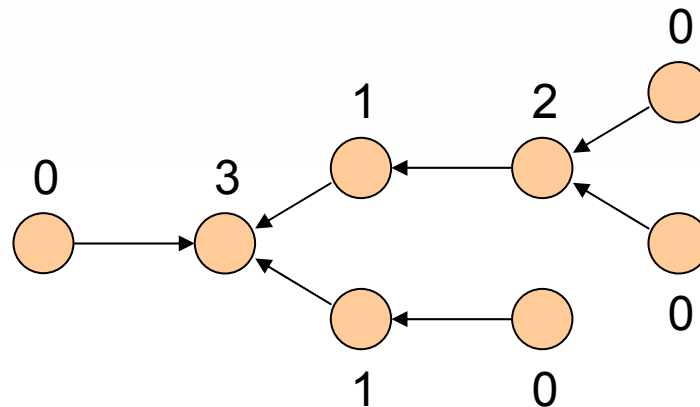


# I. Prestige

- Prestigious people
  - People who receive many positive in-links
- Structural prestige measures
  - Popularity or in-degree
  - (Restricted) input domain
  - Proximity prestige
- Structural prestige  $\neq$  social prestige (social status)
- Correlation between structural and social prestige
  - Pearson's correlation coefficient
  - Spearman's rank correlation coefficient

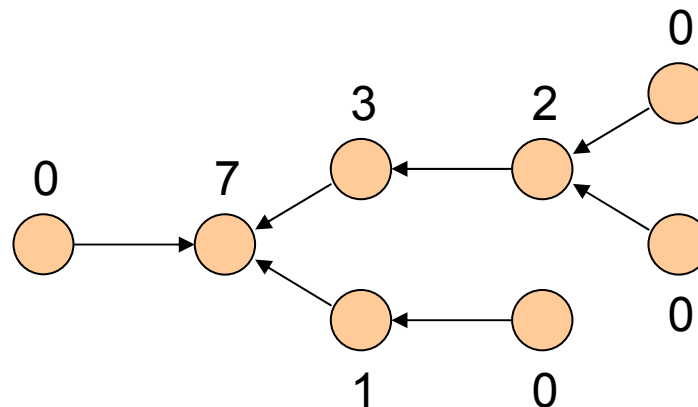


# Popularity or in-degree



# Input domain

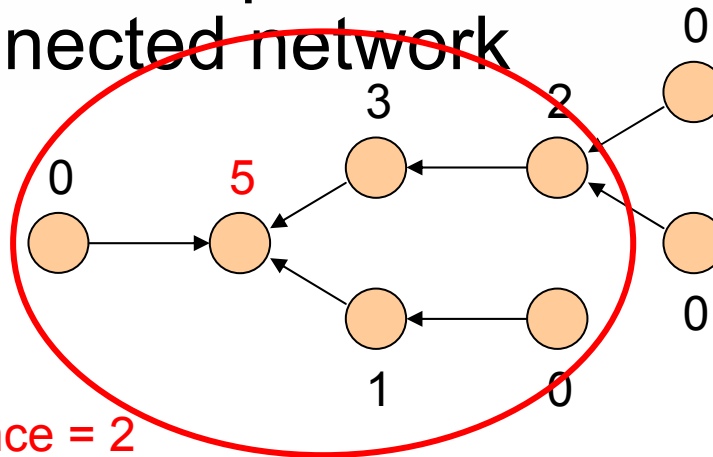
- Input domain size
  - How many nodes are path-connected to a particular node?
- Overall structure of the network is taken into account



- Problematic in a well-connected network

# Restricted input domain

- Resolves the input-domain issue in a well-connected network



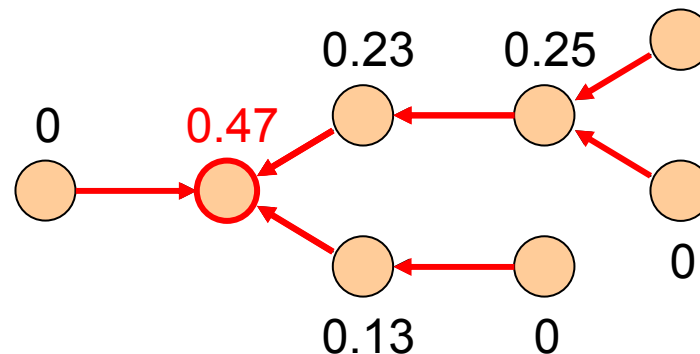
Maximum distance = 2

- Issue: the choice of the maximum distance is quite arbitrary



# Proximity prestige

- Eliminates the maximum-distance parameter
- Closer neighbors are weighted higher



$$\text{Proximity prestige} = \frac{\text{Input domain size} / \text{Number of nodes}}{\text{Average path distance to the node}} = 0.47$$

$\frac{7 / 8}{(3+3+2+2+1+1+1) / 7}$



# Structural prestige ILPnet2 dataset top 25

Input degree	28	MUGGLETON, S. H.	152	LAMMA, E.	0.082030307	RAEDT, L. D.
	21	RAEDT, L. D.	152	RIGUZZI, F.	0.077044151	DZEROSKI, S.
	20	DZEROSKI, S.	152	PEREIRA, L. M.	0.068453862	LAVRAC, N.
	17	LAVRAC, N.	152	RAMON, J.	0.066777042	MUGGLETON, S. H.
	17	BLOCKEEL, H.	152	FLACH, P. A.	0.064946309	ADE, H.
	12	FLACH, P. A.	152	LAVRAC, N.	0.06462585	BRUYNNOOGHE, M.
	12	SRINIVASAN, A.	152	STRUYF, J.	0.063683172	LAER, W. V.
	11	GYIMOTHY, T.	152	BLOCKEEL, H.	0.060918631	TODOROVSKI, L.
	10	JACOBS, N.	152	DEHASPE, L.	0.057783113	FLACH, P. A.
	10	BERGADANO, F.	152	LAER, W. V.	0.054504505	SRINIVASAN, A.
	9	WROBEL, S.	152	BRUYNNOOGHE, M.	0.054346497	GAMBERGER, D.
	9	STEPANKOVA, O.	152	DZEROSKI, S.	0.052812523	SABLON, G.
	9	ITOH, H.	152	RAEDT, L. D.	0.051974229	DEHASPE, L.
	9	ADE, H.	152	GAMBERGER, D.	0.051837094	BLOCKEEL, H.
	8	KING, R. D.	152	LACHICHE, N.	0.048245614	KING, R. D.
	8	OHWADA, H.	152	TODOROVSKI, L.	0.048015873	STERNBERG, M. J. E.
	8	BRUYNNOOGHE, M.	152	KAKAS, A. C.	0.047743034	KAKAS, A. C.
	8	BOSTROM, H.	152	JOVANOSKI, V.	0.047283414	LACHICHE, N.
	8	KRAMER, S.	152	TURNERY, P.	0.044957113	JOVANOSKI, V.
	8	FURUKAWA, K.	152	ADE, H.	0.044957113	TURNERY, P.
	8	CSIRIK, J.	152	DIMOPOULOS, Y.	0.043609897	RAMON, J.
	7	HORVATH, T.	152	SABLON, G.	0.043226091	STRUYF, J.
	7	ESPOSITO, F.	77	KING, R. D.	0.040507749	RIGUZZI, F.
	7	SHOUDAI, T.	77	MUGGLETON, S. H.	0.040341393	DIMOPOULOS, Y.
	7	DEHASPE, L.	77	SRINIVASAN, A.	0.035082604	LAMMA, E.

Unrestricted input domain size

Proximity prestige



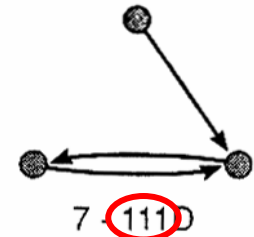
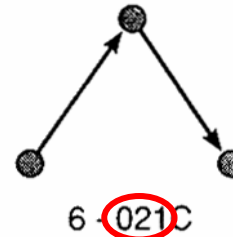
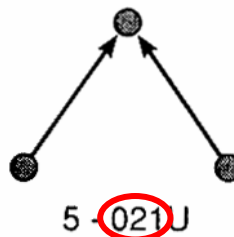


## II. Ranking

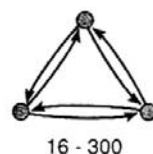
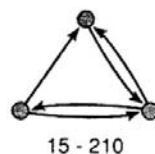
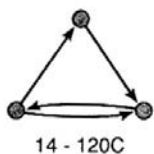
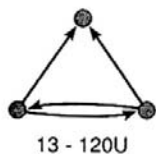
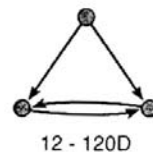
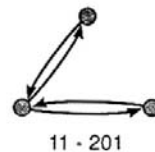
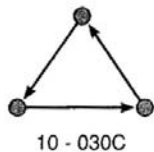
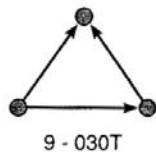
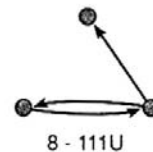
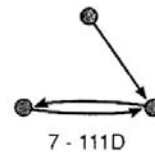
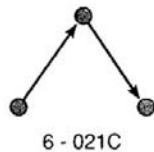
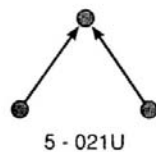
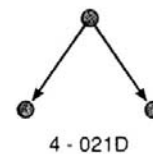
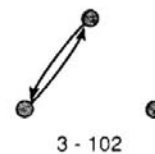
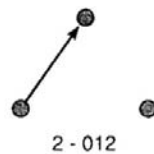
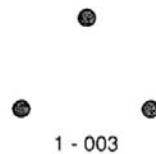
- We discuss techniques to extract discrete ranks from social relations
- Triad analysis helps us determine if our network is biased towards...
  - Unrelated clusters (cluster = clique)
  - Ranked clusters
  - Hierarchical clusters
- Recipes for determining the hierarchy
  - Acyclic decomposition
  - Symmetric-acyclic decomposition

# Triad analysis

- Triads
  - Atomic network structures (local)
  - 16 different types
  - M-A-N naming convention
    - **M**utual positive
    - **A**symmetric
    - **N**ull





# All 16 types of triads



# Triad census

- 6 balance-theoretic models

– Balance	Restricted to		
– Clusterability	Unrelated clusters		
– Ranked clusters	Ranked clusters		
– Transitivity	Less and less restricted		
– Hierarchical clusters	Hierarchical clusters		
– (Theoretic model)	Unrestricted		

- Triad census:** triads found in the network, arranged by the balance-theoretic model to which they belong



# Triad census ILPnet2 dataset

Type	Number of triads (ni)	Expected (ei)	(ni-ei)/ei	Model
3 - 102	247225	1292.72	190.24	Balance
16 - 300	112	0.00	1539118270.84	Balance
1 - 003	33404551	33159112.00	0.01	Clusterability
4 - 021D	36	1292.72	-0.97	Ranked Clusters
5 - 021U	1176	1292.72	0.09	Ranked Clusters
9 - 030T	39	9.32	3.18	Ranked Clusters
12 - 120D	91	0.02	5415.95	Ranked Clusters
13 - 120U	83	0.02	4939.74	Ranked Clusters
2 - 012	228351	717207.24	-0.68	Transitivity
14 - 120C	1	0.03	28.76	Hierarchical Clusters
15 - 210	64	0.00	528411.66	Hierarchical Clusters
6 - 021C	182	2585.44	-0.93	Forbidden
7 - 111D	719	9.32	76.14	Forbidden
8 - 111U	63	9.32	5.76	Forbidden
10 - 030C	0	3.11	-1.00	Forbidden
11 - 201	121	0.02	7201.76	Forbidden

Chi-Square: 172464018511.5997\*\*\*

7 cells (43.75%) have expected frequencies less than 5.

The minimum expected cell frequency is 0.00.

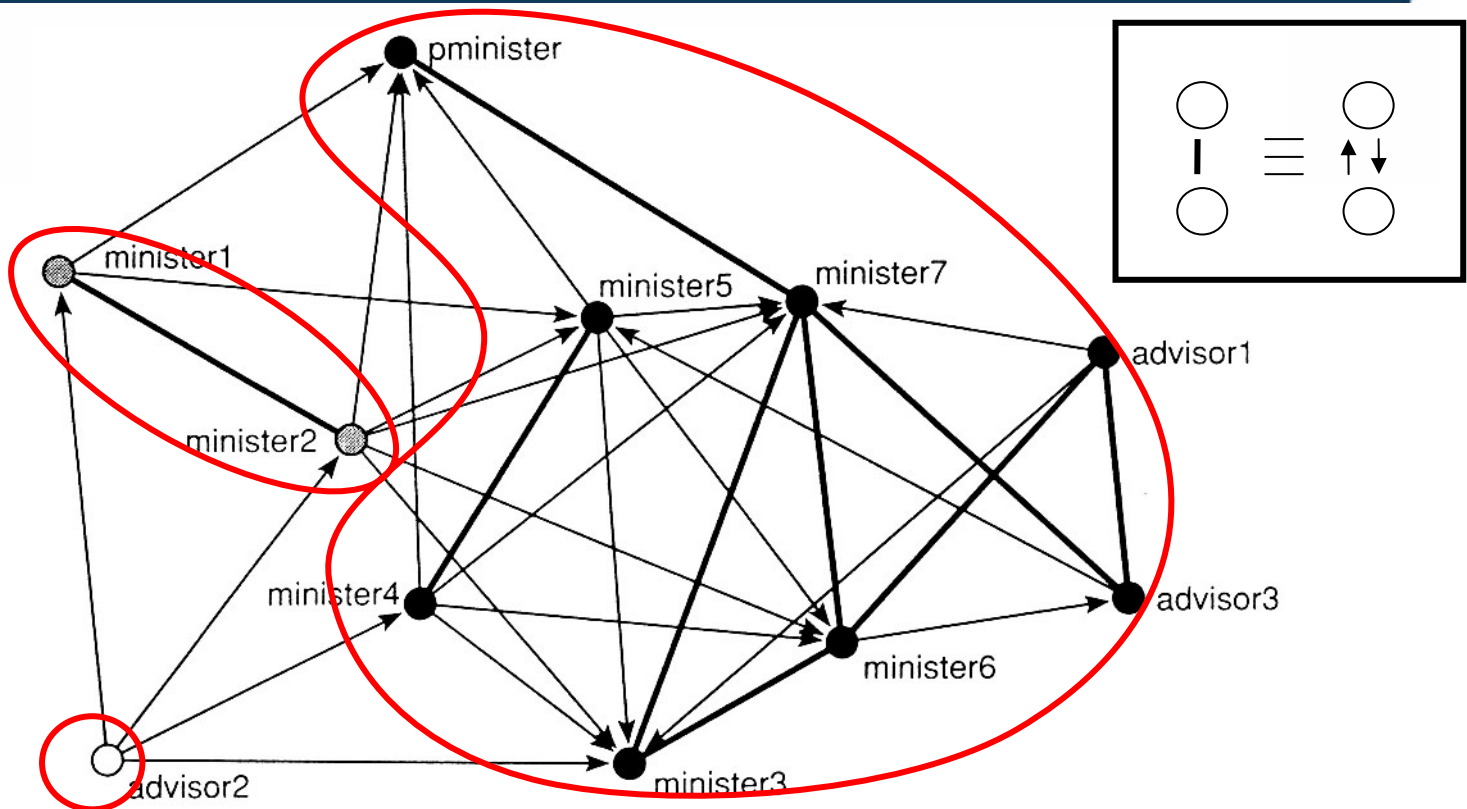


# Acyclic decomposition

- Cyclic networks (strong components) are clusters of equals
- Acyclic networks perfectly reflect hierarchy
- Recipe
  - Partition the network into strong components (i.e. clusters of equals)
  - Create a new network in which each node represents one cluster
  - Compute the maximum depth of each node to determine the hierarchy

# Acyclic decomposition

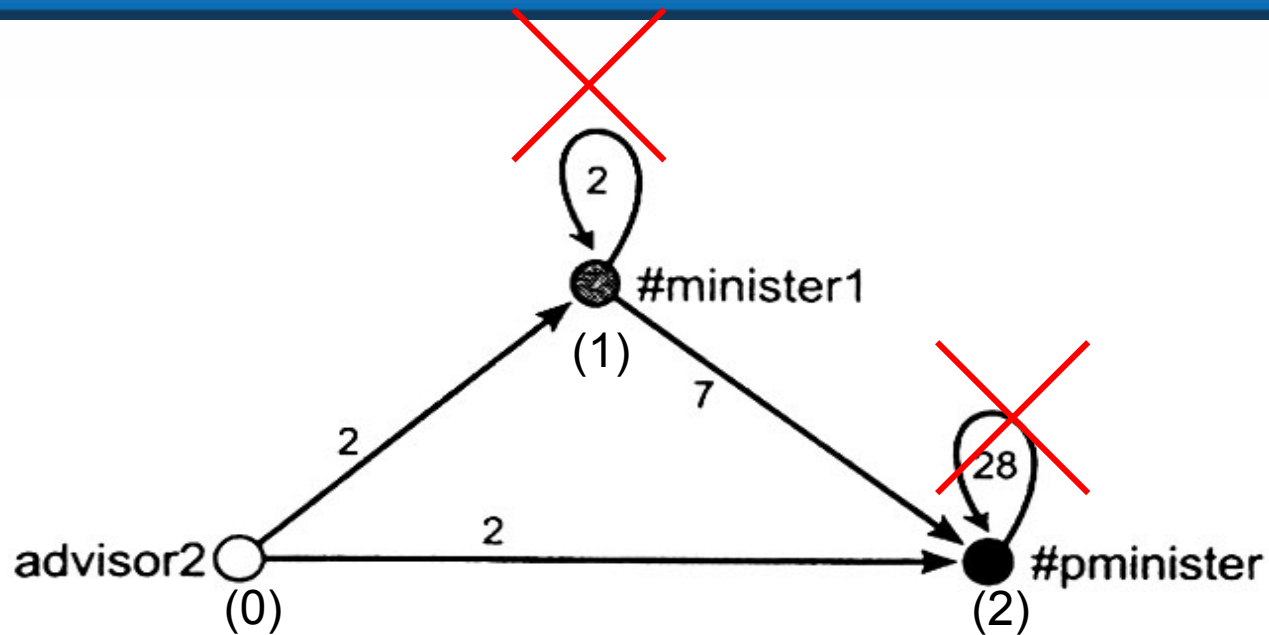
## An example





# Acyclic decomposition

## An example



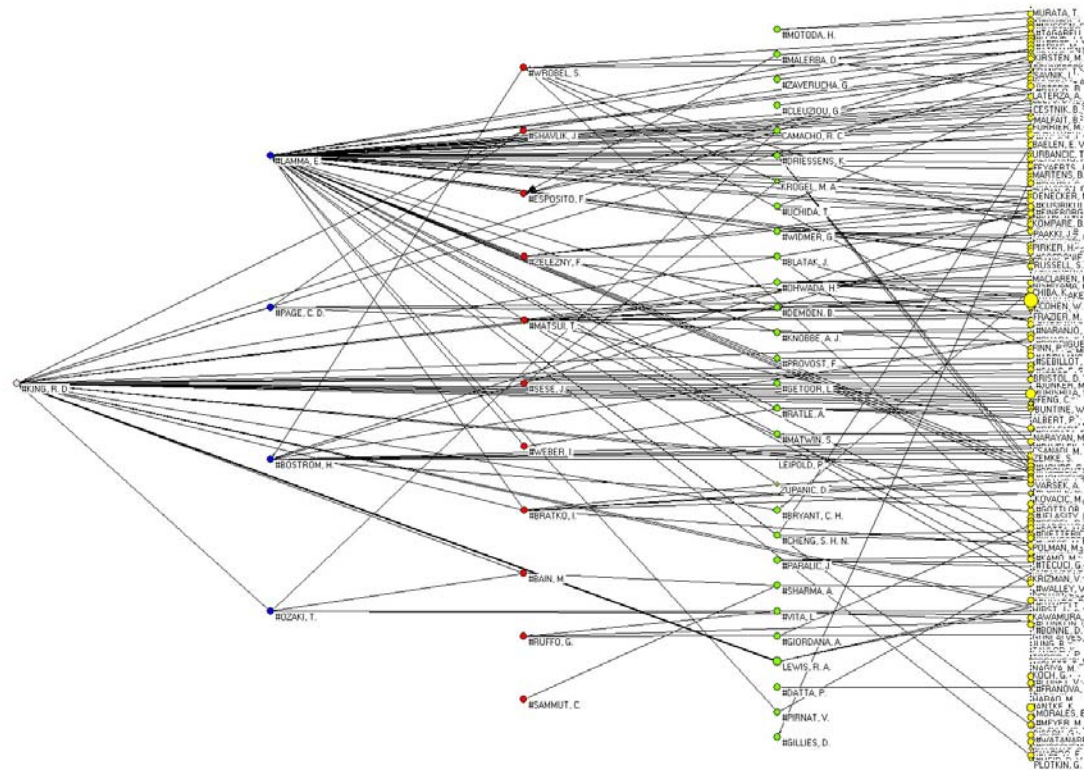
The maximum depth of a node determines its position in the hierarchy

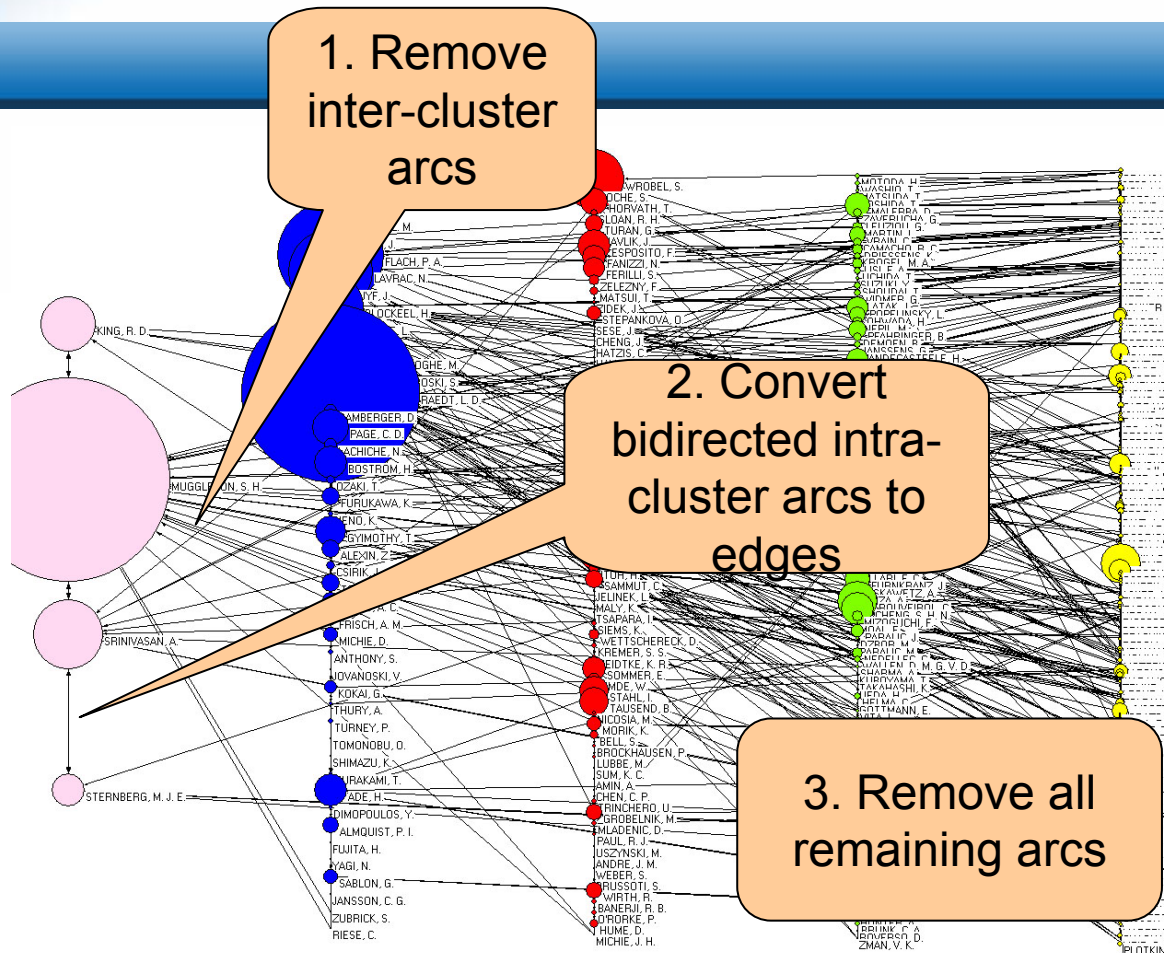


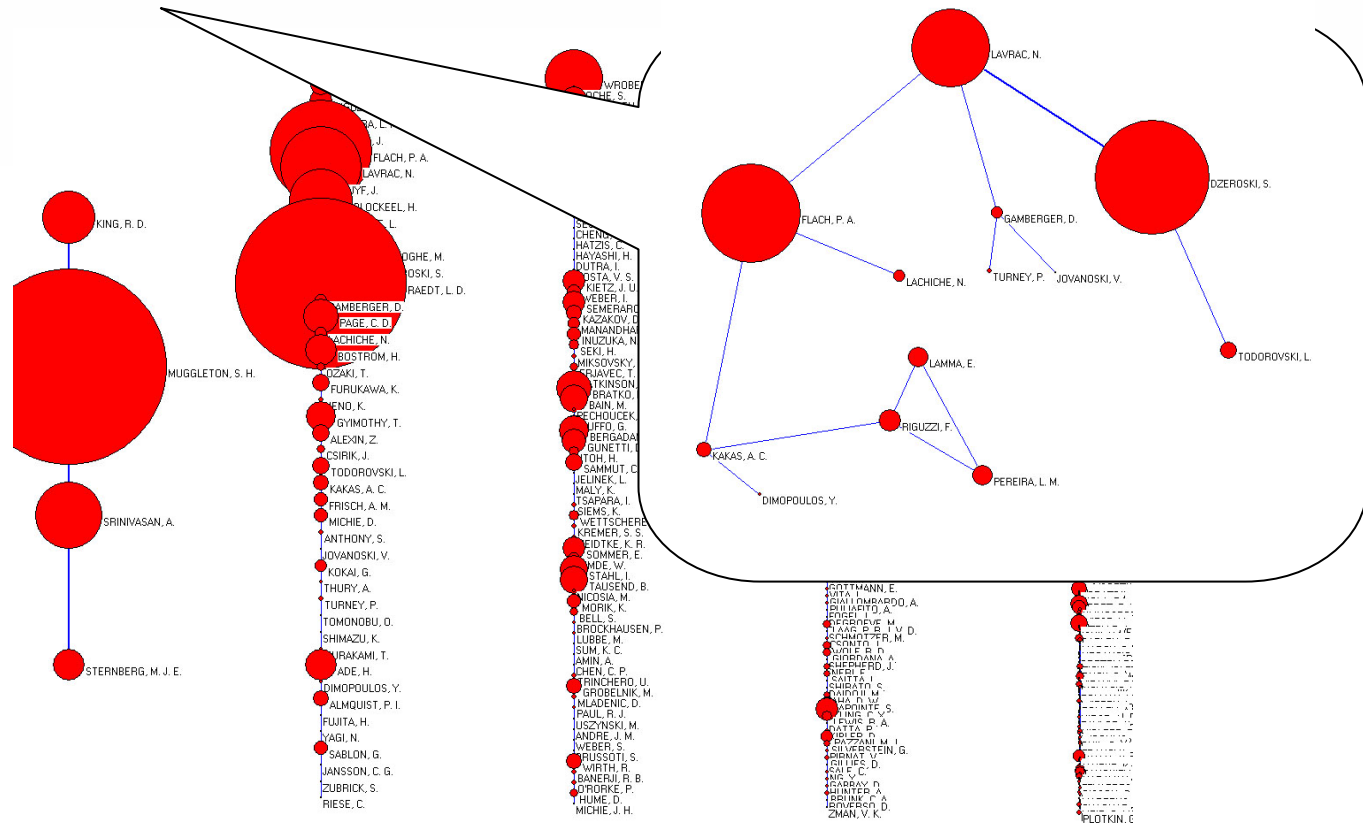


# Acyclic decomposition ILPnet2 dataset











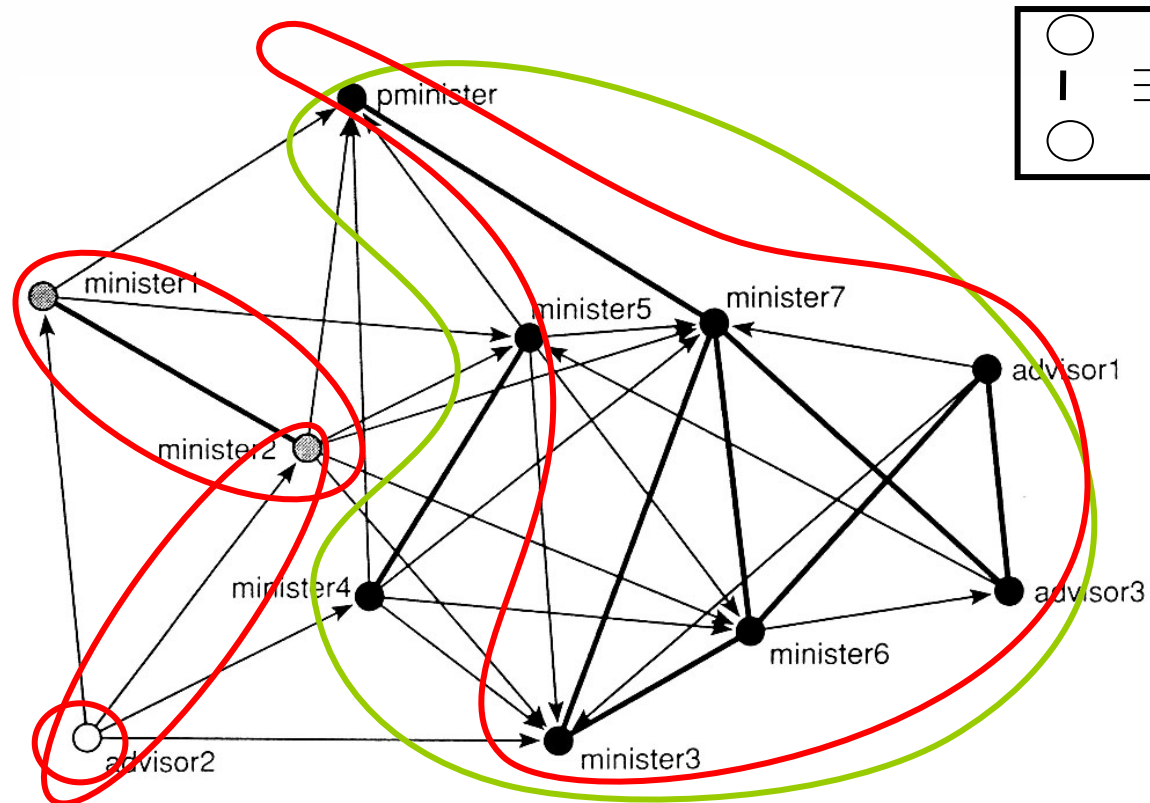
# Symmetric-acyclic decomposition

- Strong components are not strict enough to detect clusters in the triad-census sense
- Symmetric-acyclic decomposition extracts clusters of vertices that are connected both ways
- After the clusters are identified, we can follow the same steps as in acyclic decomposition to determine the hierarchy



# Symmetric-acyclic decomp.

## An example





# Reference

- Batagelj V., Mrvar A., de Nooy W. (2004):  
**Exploratory Network Analysis with Pajek.**  
Cambridge University Press
- Some figures used in the presentation are taken from this book
- Most of the presentations are taken from presentation of ILPNet2 Social Network Analysis Project of Sergeja Sabo, David Fabjan and Miha Grčar