Decision Analysis
Part 2:
Decision Trees

Working Example
Decision table (Payoff matrix)

<p>|</p>
<table>
<thead>
<tr>
<th>alternative</th>
<th>status quo</th>
<th>extend</th>
<th>right</th>
<th>cope a b</th>
</tr>
</thead>
<tbody>
<tr>
<td>decreased sales</td>
<td>28</td>
<td>24</td>
<td>16</td>
<td>30</td>
</tr>
<tr>
<td>increased sales</td>
<td>30</td>
<td>42</td>
<td>44</td>
<td>34</td>
</tr>
</tbody>
</table>

Working Example
Equivalent decision tree

events (states) expected profit
influ. dec. decreased sales (0.25) 28
influ. dec. increased sales (0.75) 30
decl. dec. decreased sales (0.25) 24
decl. dec. increased sales (0.75) 42
decl. incl. decreased sales (0.25) 16
decl. incl. increased sales (0.75) 44
build decreased sales (0.25) 30
build increased sales (0.75) 34
Decision Tree

Different from decision trees used in Machine Learning
- different types of nodes
- always drawn horizontally, from left to right
- "hand-crafted", not learned from data

Decision tree represents the decision problem in terms of
chains of consecutive decisions and chance events.
Time proceeds from left to right.
Uncertainties associated with chance events are modelled by
probabilities.

Components of Decision Trees

Alternative 1
Decision node:
represents alternatives

Alternative 2

Event 1 (Prob 1)
Chance Node:
represents events (states of nature)

Event 2 (Prob 2)

Outcome (Value)
Terminal (End) Node:
represents consequences of decisions

Solving Decision Trees

From right to left:

EV
 Alternative 1 EV
 Alternative 2 EV

EV
 Event 1 (Prob 1) EV
 Event 2 (Prob 2) EV

EV
 Outcome (Value) EV = Value

EV

EV = max, EVi [maximize profit] or
EV = min, EVi [minimise losses]
### Solved Decision Tree

- **alternatives**
  - status quo
  - extend
  - build
  - cooperate

- **events**
  - decreased sales (0.25)
  - increased sales (0.75)

- **expected profit**
  - decreased sales (0.25): 28
  - increased sales (0.75): 29.5

### Decision Tree Development

1. Place decision and chance nodes in a logical time order
2. Independent chance nodes can be placed in any order
3. Estimate probabilities of all chance events
4. The sum of probabilities in a chance node must be 1
5. In terminal nodes, specify consequences by a single performance measure, e.g.:
   - money,
   - aggregate utility or
   - results of a multiple criteria analysis

### Common Mistakes

1. Decision and chance nodes are in wrong order:
   Only chance nodes whose results are known at the time of decision can precede a decision node
2. Incorrect derivation of chance probabilities:
   Chance probabilities depend on each other and decisions made
3. Chance events with probability 0 can be left out
4. When solving the tree:
   Maximising instead of minimising, or vice versa
Example 1: Oilco

Mobon Oil Company has a lease on an offshore oil site. The lease is about to expire and they are faced with either developing the field or selling the lease to Excel Oil Co. for $50,000. It costs approximately $100,000 to drill a well. There is a 45% chance that the well is dry, a 45% chance that the well will have a minor strike and a 10% chance that they will have a major strike. For a typical minor strike the revenues average $300,000. If the strike is major the revenues average $700,000. What should Mobon do?

Recommendation: Drill!

Example 2: Sequential Decision
Other Important Concepts

1. Value of Perfect Information
2. Risk Profile

Value of Perfect Information

Example

• Oilco must determine whether or not to drill for oil in the South China Sea. It costs $100,000 to drill for oil and if oil is found the value of the oil is estimated to be $600,000. At present, Oilco believes there is a 45% chance that the field contains oil. What should Oilco do?
• What is the value of perfect information (knowledge of whether the field contains oil) to Oilco?
Ordinary Decision Tree

\[
egin{aligned}
0.55 \times 0 &= 0 \\
0.45 \times 600K &= 270K \\
Total &= 3270K \\
\end{aligned}
\]

\[
\begin{aligned}
Drill &\rightarrow 3270K \\
0.55 \text{ Dry} &\rightarrow 0 \\
0.45 \text{ Oil} &\rightarrow 270K \\
\end{aligned}
\]

\[
\begin{aligned}
\text{Net Drill} &\rightarrow 0 \\
\$600K &\rightarrow 0 \\
\$0 &\rightarrow 0 \\
\end{aligned}
\]

Recommendation: Drill!

Value of Perfect Information

Exchange decision and event node:

\[
\begin{aligned}
500K &\rightarrow 500K \\
0.45 \text{ Oil} &\rightarrow 0 \\
0.55 \text{ no Oil} &\rightarrow 0 \\
\end{aligned}
\]

\[
\begin{aligned}
\$225 &\rightarrow 225 \\
0.45 \text{ Oil} &\rightarrow 0 \\
0.55 \text{ no Oil} &\rightarrow 0 \\
\end{aligned}
\]

\[
\begin{aligned}
\text{Drill} &\rightarrow 225 \\
0 \text{ Oil} &\rightarrow 0 \\
0 \text{ no Oil} &\rightarrow 0 \\
\end{aligned}
\]

\[
\begin{aligned}
\text{Net Drill} &\rightarrow 0 \\
\$50K &\rightarrow 0 \\
\$0 &\rightarrow 0 \\
\end{aligned}
\]

Value of Perfect Information: $225 - $170 - $55
Solved Decision Tree

Risk Profile

Cumulative Risk Profile
Decision Tree Software

Decision Analysis Software
http://www.decisionanalysisonline.com/software.html

Decision Tree Software

Add-ins for Microsoft Excel:
1. TreePlan: http://www.treeplan.com/
2. DecisionTree: http://www.pal-sade-europe.com/decisiontree/

Decision - Tree Development Programs:
1. TreeAge Pro (DATA): http://www.treeage.com/
2. DecisionPro: http://www.venturedww.com/
3. DPL: http://www.synoptics.com/dpl/
A Typical Application in Medicine

A Cost-Benefit Analysis of Testing for Influenza A in High-Risk Adults

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Aims

1. To evaluate the cost-effectiveness of testing for influenza A in high-risk adults.
2. To determine the effectiveness of different testing strategies on reducing the spread of influenza A.

Methods

A randomized controlled trial was conducted involving high-risk adults. Participants were randomized into two groups: one group received routine testing for influenza A, while the other group received no testing. The outcomes measured were the incidence of influenza A and the hospitalization rates among participants.

Results

Routine testing for influenza A significantly reduced the incidence of influenza A compared to the control group (p < 0.05). However, there was no significant difference in hospitalization rates between the two groups.

Conclusion

Routine testing for influenza A in high-risk adults is an effective strategy for reducing the incidence of influenza A. Further studies are needed to determine the long-term effects of routine testing on the spread of influenza A.

A Typical Application in Medicine

Questions

Compare decision tables with decision trees.

What do decision trees facilitate that decision tables don’t?

Identify limitations and/or shortcomings of decision trees.

Identify types of decision problems suitable for the application of decision trees.
Exercise 1
Consider the decision tree:

1. Solve the tree for tomorrow’s $p$
2. Do sensitivity analysis
3. Take the risk profile of your decision

Exercise 2
For the decision tree shown in the slide “Example 2: Sequential Decision” (Introduce Product):

1. Do sensitivity analysis with respect to $p(\text{competitor})$
2. Find the risk profile of alternative introduce.

Tree Development Exercise (1/3)

Service station problem:
• You are the owner of a service station on an intercity road. You have heard a rumour that the road may be upgraded or diverted along a different route. What do you do? What information will you need? How do you formulate a decision model?
• Think before reading next slide, structure your own decision. You will need to specify an objective, identify alternatives available to you as the service station owner, and identify the uncertainties involved in this decision situation together with the possible events.
Tree Development Exercise (2/3)

Possible answers to the questions from the previous slide:
Objective: maximise the value of service station investment
Alternatives:
- status quo
- sell
- extend
Events:
- unaltered \( (p=0.5) \)
- upgrade \( (p=0.3) \)
- divert \( (p=0.2) \)

Tree Development Exercise (3/3)

Proceed as follows:
1. Define decision table (include consequences)
2. Convert decision table to decision tree
3. Calculate EV and identify the best alternative
4. Do sensitivity analysis with respect to \( p(\text{unaltered}) \)
   [which problem do you encounter here?]
5. Find the risk profile of the best alternative