Decision Analysis
Part 1
Decision Analysis and Decision Tables

Decision Analysis, Part 1

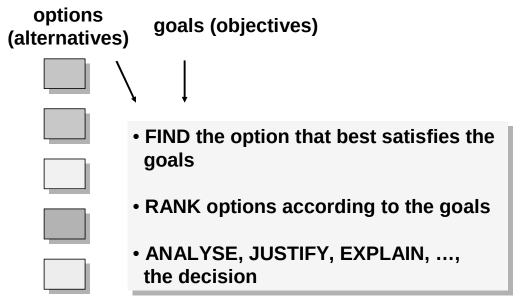
- Introduction to Decision Analysis
  - Concepts: modelling, evaluation, analysis
  - Decision Problem-Solving: Stages
  - Relation of DA to some other Disciplines
- Decision-Making under Uncertainty
  - Decision-Making under Strict Uncertainty
    - Decision Table
    - Various Decision Criteria
  - Decision-Making under Risk
    - Expected Value
    - Sensitivity Analysis

Decision Analysis

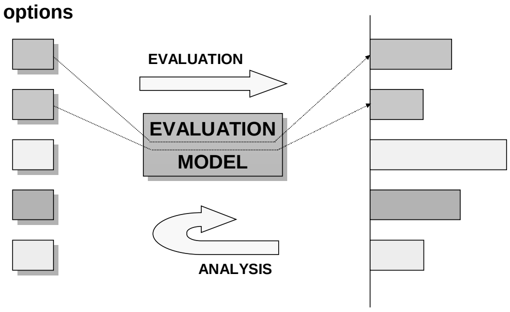
Decision Analysis: Applied Decision Theory

Provides a framework for analyzing decision problems by
- structuring and breaking them down into more manageable parts,
- explicitly considering the:
  - possible alternatives,
  - available information
  - uncertainties involved, and
  - relevant preferences
- combining these to arrive at optimal (or "good") decisions

Decision-Making Problem

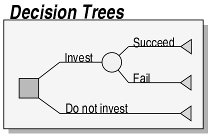


Evaluation Models

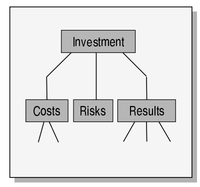


Types of Models in Decision Analysis

Decision Trees

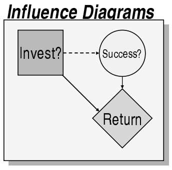


Multi-Attribute Utility Models

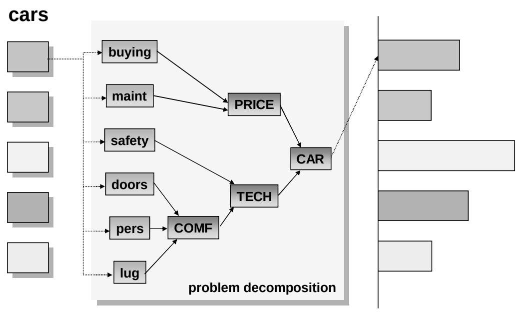


Analytic Hierarchy Process

Influence Diagrams



Multi-Attribute Models



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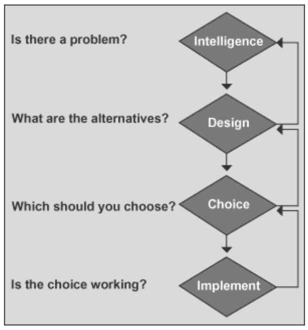
Decision-Making Process



Source: Decision Analysis – A Tool to Deal with Uncertainty, http://www.dmreview.com/article_sub.cfm?articleId=6935

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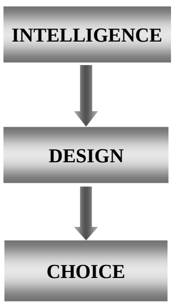
Decision-Making Process



- Fact Finding
- Problem/Opportunity Sensing
- Analysis/Exploration

- Formulation of Solutions
- Generation of Alternatives
- Modelling/Simulation

- Alternative Selection
- Goal Maximization
- Decision Making
- Implementation

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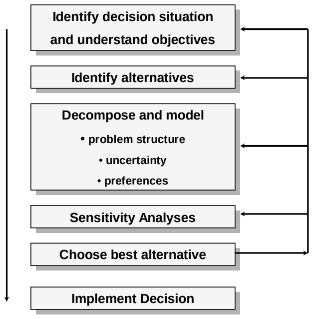
The Decision Analysis Process



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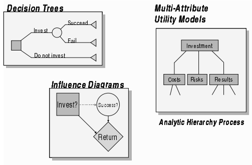
Decision Analysis: Related Disciplines

⇑ DSS, GDSS
- Group Decision Process

⇐ OR/MS
- Multi-Criteria Optimisation
- Risk Analysis and Simulation
- Bayesian Networks
- Markov Modelling



ES, ML ⇒
- Qualitative Multi-Attribute Models

- *Decision Tables*
- *Outranking*
- *Multi-Criteria Partial Ordering*

- Mathematical/Algebraic/Statistical Modelling
- Accounting / Financial Modelling

⇓ Modelling

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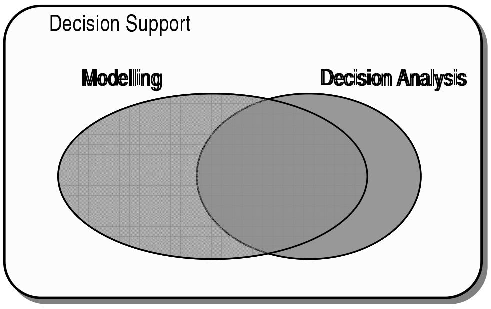
Decision Support and Decision Modeling



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Decision-Making under Uncertainty

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Decision-Making Problem

Suppose that one must choose between several *uncertain alternatives*.

Given:

- *Alternatives*;
- The *consequences* of choosing each alternative, described with a *single* number, e.g. profit / loss in € or aggregated value.

Task: Which alternative to choose?

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Decision Table

Decision-Making under Strict Uncertainty

State of the world (Event)	Value of alternatives $1 \dots m$		
θ	a_1	...	a_m
θ_1	y_{11}	...	y_{1m}
:	:		:
θ_n	y_{n1}	...	y_{nm}

Decision-Making under Risk

State of the world (Event)	Probability that θ will happen	Value of alternatives $1 \dots m$		
θ	$P(\theta)$	a_1	...	a_m
θ_1	$p(\theta_1)$	y_{11}	...	y_{1m}
:	:	:		:
θ_n	$p(\theta_n)$	y_{n1}	...	y_{nm}

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Working Example

A manufacturing company, faced with a possible increase in demand for its product, considers the following:

Alternatives:

1. *status quo*: no change
2. *extend*: extending their production line buying a new machine
3. *build*: building a new production hall with new equipment
4. *cooperate*: finding additional business parters for production

Uncertainty involved:
Market reaction: after the decision, the sales can *increase* or *decrease*.

Consequences:
Expected profit, shown in decision table on the next slide

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Working Example

Decision table

		alternative			
		status quo	*extend*	*build*	*cooperate*
states	*decreased sales*	28	24	16	30
	increased sales	30	42	44	34

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Decision-Making under Strict Uncertainty

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Decision Criteria

- Dominance
- Pessimistic (Maximin, Wald's)
- Optimistic (Maximax)
- Hurwicz's
- Laplace's
- Minimax Regret

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Dominance

- Choose the alternative with best consequences in all states of the world.
- Such alternative is seldom found.

		alternative			
		status quo	*extend*	*build*	*cooperate*
states	*decreased sales*	28	24	16	30
	increased sales	30	42	44	34

No dominant alternatives in this case

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Pessimistic Criterion (Wald's, Maximin)

- Each alternative is represented by its *worst* possible consequence.
- According to these, the alternative with the *best* worst case is chosen.

		alternative			
		status quo	*extend*	*build*	*cooperate*
states	*decreased sales*	*28*	*24*	*16*	*30*
	increased sales	*30*	*42*	*44*	*34*
Pessimist		**28**	**24**	**16**	**<u>30</u>**

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Optimistic Criterion (Maximax)

- Each alternative is represented by its *best* possible consequence.
- The alternative for which this *best* consequence is best is chosen.

		alternative			
		status quo	*extend*	*build*	*cooperate*
states	*decreased sales*	*28*	*24*	*16*	*30*
	increased sales	*30*	*42*	*44*	*34*
Optimist		**30**	**42**	**<u>44</u>**	**34**

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Hurwicz's Criterion

- Introduce a parameter $d \in [0,1]$.
- Combine Optimistic and Pessimistic criteria so that

$$u_h = du_o + (1-d)u_p$$

		alternative			
		status quo	*extend*	*build*	*cooperate*
states	*decreased sales*	*28*	*24*	*16*	*30*
	increased sales	*30*	*42*	*44*	*34*
Pessimist		**28**	**24**	**16**	**<u>30</u>**
Optimist		**30**	**42**	**<u>44</u>**	**34**
Hurwiz (*d*=0,3)		**28,6**	**29,4**	**24,4**	**<u>31,2</u>**

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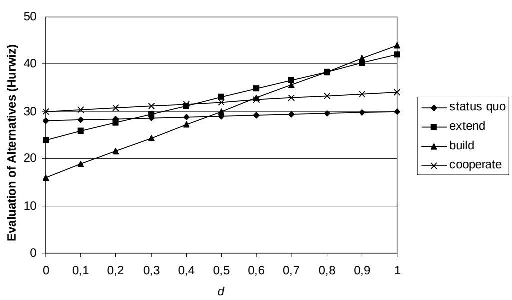
Hurwicz's Criterion



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Laplace's Criterion

- Consider all states (events) equally likely,
- thus, consider the *average* of outcomes for each alternative.

		alternative			
		status quo	*extend*	*build*	*cooperate*
states	*decreased sales*	*28*	*24*	*16*	*30*
	increased sales	*30*	*42*	*44*	*34*
Laplace		**29**	**33**	**30**	**32**

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Minimax Regret

The **regret** r_{ij} for the alternative a_j in state θ_i is equal to the difference between the best alternative in given state θ_i and a_j: $r_{ij} = \max_{k=1}^{m}\{y_{ik}\} - y_{ij}$

Choose the alternative having the least maximum regret.

		alternative			
		status quo	*extend*	*build*	*cooperate*
States	*decreased sales*	30–28=2	30–24=6	30–16=14	30–30=0
	increased sales	44–30=14	44–42=2	44–44=0	44–34=10
Regret		**14**	**6**	**14**	**10**

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Summary

		alternative			
		status quo	*extend*	*build*	*cooperate*
states	*decreased sales*	*28*	*24*	*16*	*30*
	increased sales	*30*	*42*	*44*	*34*
Pessimist		**28**	**24**	**16**	**30**
Optimist		**30**	**42**	**44**	**34**
Hurwiz (d=0,3)		**28,6**	**29,4**	**24,4**	**31,2**
Laplace		**29**	**33**	**30**	**32**
Regret		**14**	**6**	**14**	**10**

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Questions

- If you were the manager, which alternative would you take? Why?
- Is this really the best alternative? Why? Under which circumstances it is best?
- What can you say about the *status quo* alternative? According to the analysis, when should be it taken, or should it be taken at all?

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Questions

Assess the presented decision criteria:

- Describe the prevalent characteristics of each criterion
- What do you think about the criteria:
  - Are they comprehensible?
  - Are they realistic?
  - Are they useful for practice?
  - Which is your favourite criterion?
- Is there a single "best" criterion? Which and why?

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Decision-Making under Risk

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Working Example

Now we know (or estimate) the *probablity* of states

		alternatives			
states	***probability***	*status quo*	*extend*	*build*	*cooperate*
decreased sales	25%	28	24	16	30
increased sales	75%	30	42	44	34

Decision Criteria

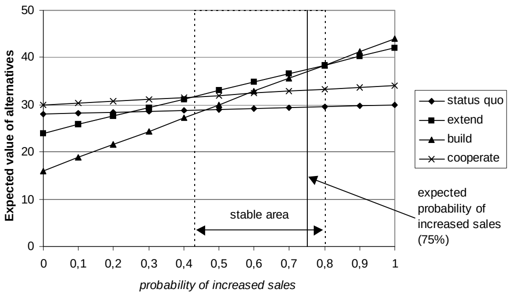
- Mode: Select the most probable state
- Expected Value (EV), Expected Monetary Value (EMV)

Expected (Monetary) Value

Maximise the expected value: $EV_i = \sum_{j=1}^{n} p(\theta_j) y_{ji}$

		alternatives			
states	***probability***	*status quo*	*extend*	*build*	*cooperate*
decreased sales	25%	28	24	16	30
increased sales	75%	30	42	44	34
Expected value		0,25×28+ 0,75×30= **29,5**	0,25×24+ 0,75×42= **<u>37,5</u>**	0,25×16+ 0,75×44= **37**	0,25×30+ 0,75×34= **33**

Sensitivity Analysis



Exercise 1

	$P(\theta)$	a_1	a_2	a_3
θ_1	2/9	8	4	20
θ_2	3/9	7	15	10
θ_3	4/9	6	5	0

Given this decision table:

- Determine which alternative is best according to all the criteria (Dominance, Pessimistic, Optimistic, Hurwiz (d=0.7), Laplace, Regret, Mode, Expected Value).
- Draw a chart evaluating the Hurwiz's criterion for $d \in [0,1]$.
- Do sensitivity analysis.

Exercise 2

Help the farmer who is deciding which crop to plant in the face of uncertain weather and resulting crop yield:

	Weather		
probability	.55	.15	30
	Normal	Drought	Rainy
Plant soybeans	$ 10	5	12
Plant corn	7	8	13

profit per acre

Exercise 3

1. Define a decision problem of your own,
2. represent it in a decision table,
3. and repeat the steps of Exercise 1

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Exercise 4

Using some decision table,
implement in spreadsheet software (such as MS Excel):

- evaluation of alternatives using all the criteria,
- drawing the chart associated with Hurwiz's criterion
- drawing the sensitivity analysis chart

Compare the two charts.

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