Numeric Relational Multi-Attribute Models in Qualitative Multi-Attribute Method DEX

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The Art of Modeling BARCELONA 2014 13th-18th July

Motivation

- DEX is a qualitative MCDM methodology, supported by implementation in DEXi
- It has been applied numerous times; ecology, financial domains, evaluation of projects, medicine, etc.
- More flexibility in methodology is needed, specifically in the direction of relational models and numerical attributes.

Overview

The DEX methodology

State of the art: General

- A qualitative multi-criteria decision making methodology
  - Developed model is a hierarchy of attributes
  - All attributes have symbolic values

1. M. Bohanec and V. Rajkovič. DEX: An expert system shell for decision support, Sistemica, vol. 1(1), pp. 145-157, 1990.
2. M. Bohanec, DEXi: A program for multi-attribute decision making. `http://kt.ijs.si/MarkoBohanec/dexi.html`, June 2013.
3. M. Bohanec, V. Rajkovič, I. Bratko, B. Zupan, M. ?nidar?i?. DEX methodology: Three decades of qualitative multi-attribute modelling. Informatica 37, 49-54, 2013.

Formalization - DEX

Attribute

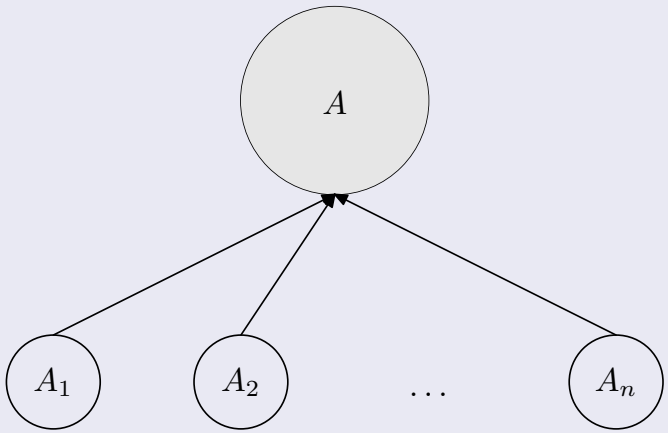
Attribute A:

- most basic building block in the methodology.
- $D(A)$ is the domain: finite array of qualitative - symbolic values.

- For example:
  - $D(A_1) = [low, medium, high]$
  - $D(A_2) = [yes, no]$
  - $D(A_3) = [2\_or\_less, 3, 4, more\_than\_4]$

Formalization - DEX

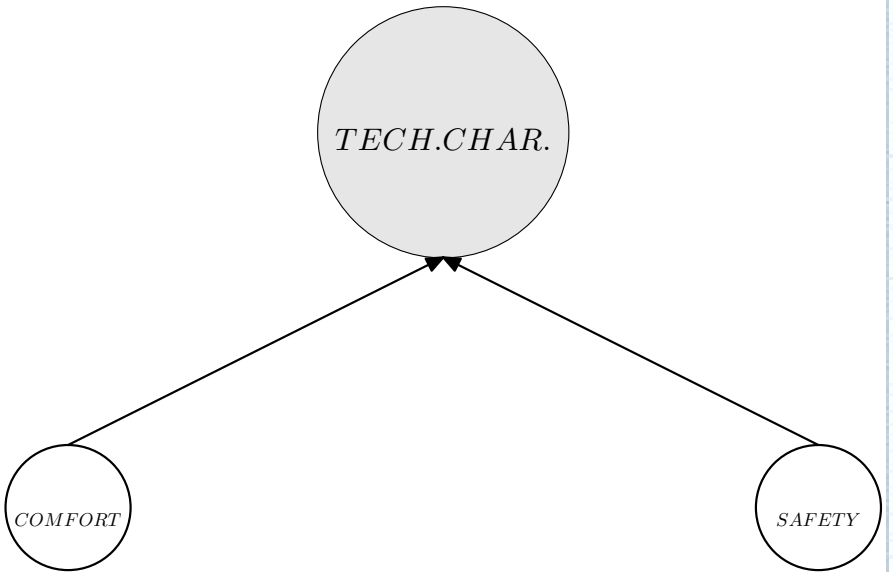
Aggregated attribute



- Aggregated attribute A:
  - logical combination of lower level attributes...
  - ..., into a higher level concept.
- Total aggregation function (defined by a table) $F_A : D(A_1) \times D(A_2) \times \ldots \times D(A_n) \rightarrow I(D(A))$.

Formalization - DEX

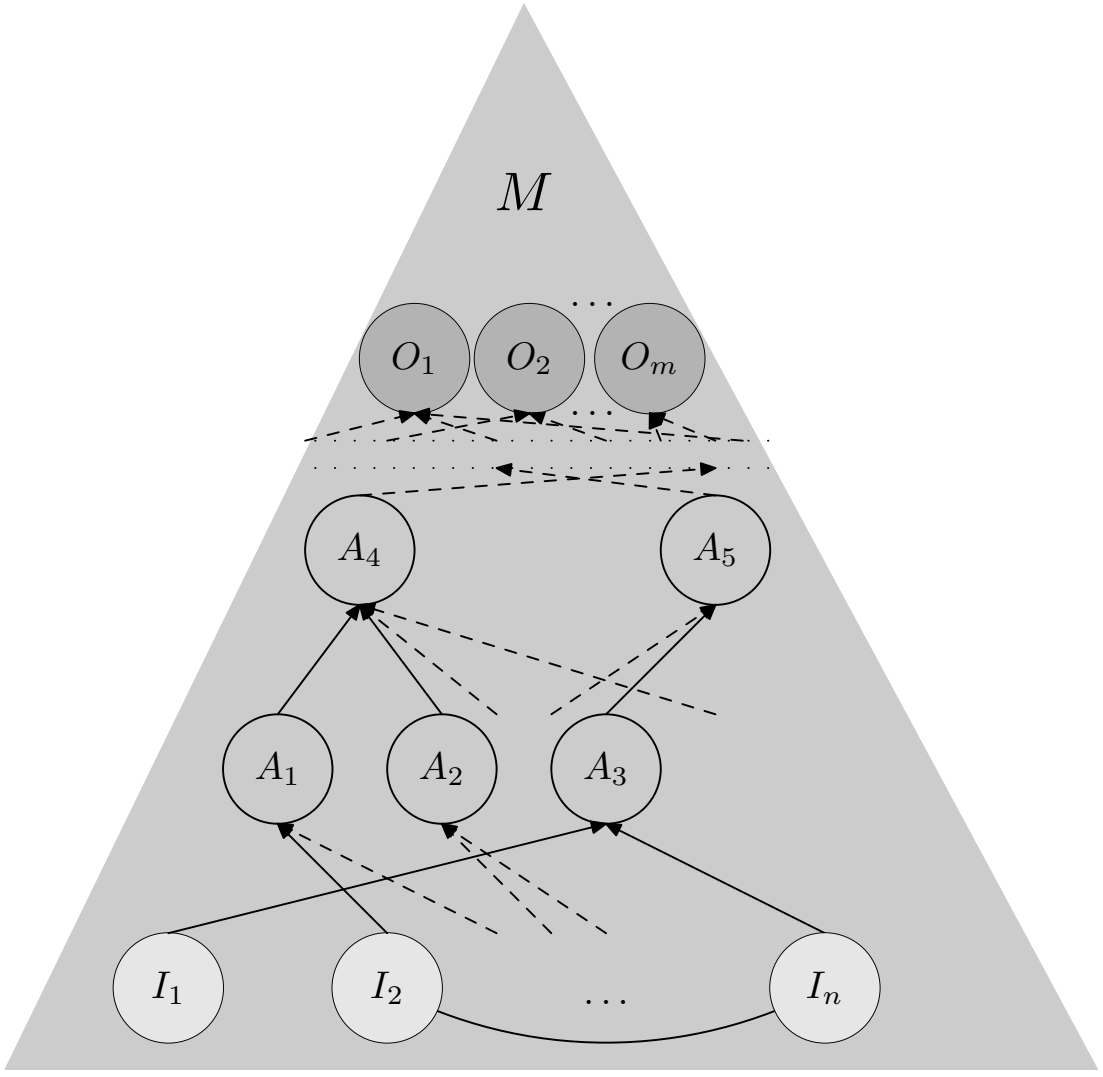
Aggregated attribute - Function example



	COMFORT	SAFETY	TECH.CHAR.
1	small	small	**bad**
2	small	medium	**bad**
3	small	high	**bad**
4	medium	small	**bad**
5	medium	medium	**acc**
6	medium	high	**good**
7	high	small	**bad**
8	high	medium	**good**
9	high	high	**exc**

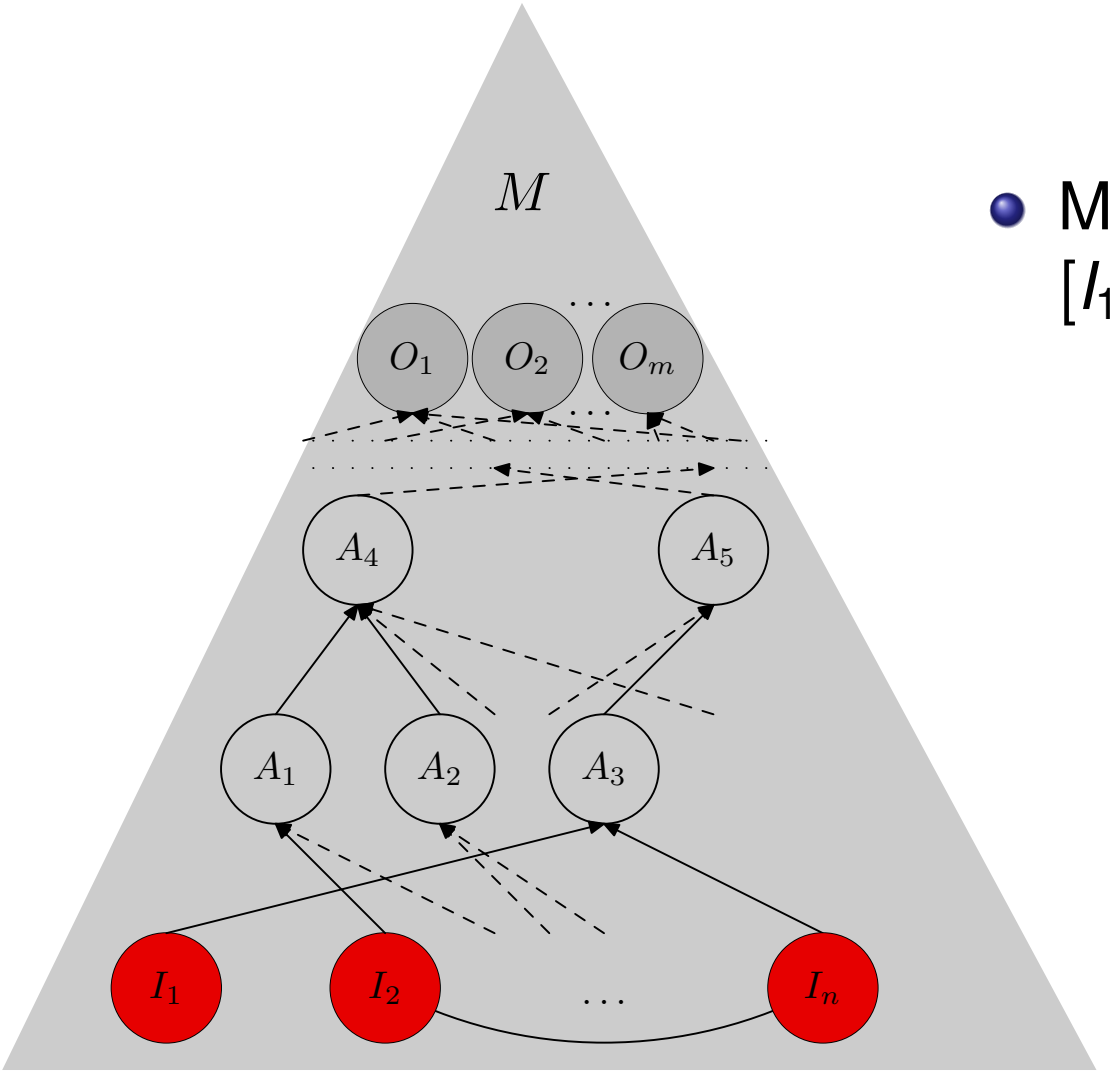
Formalization - DEX

Model



Formalization - DEX

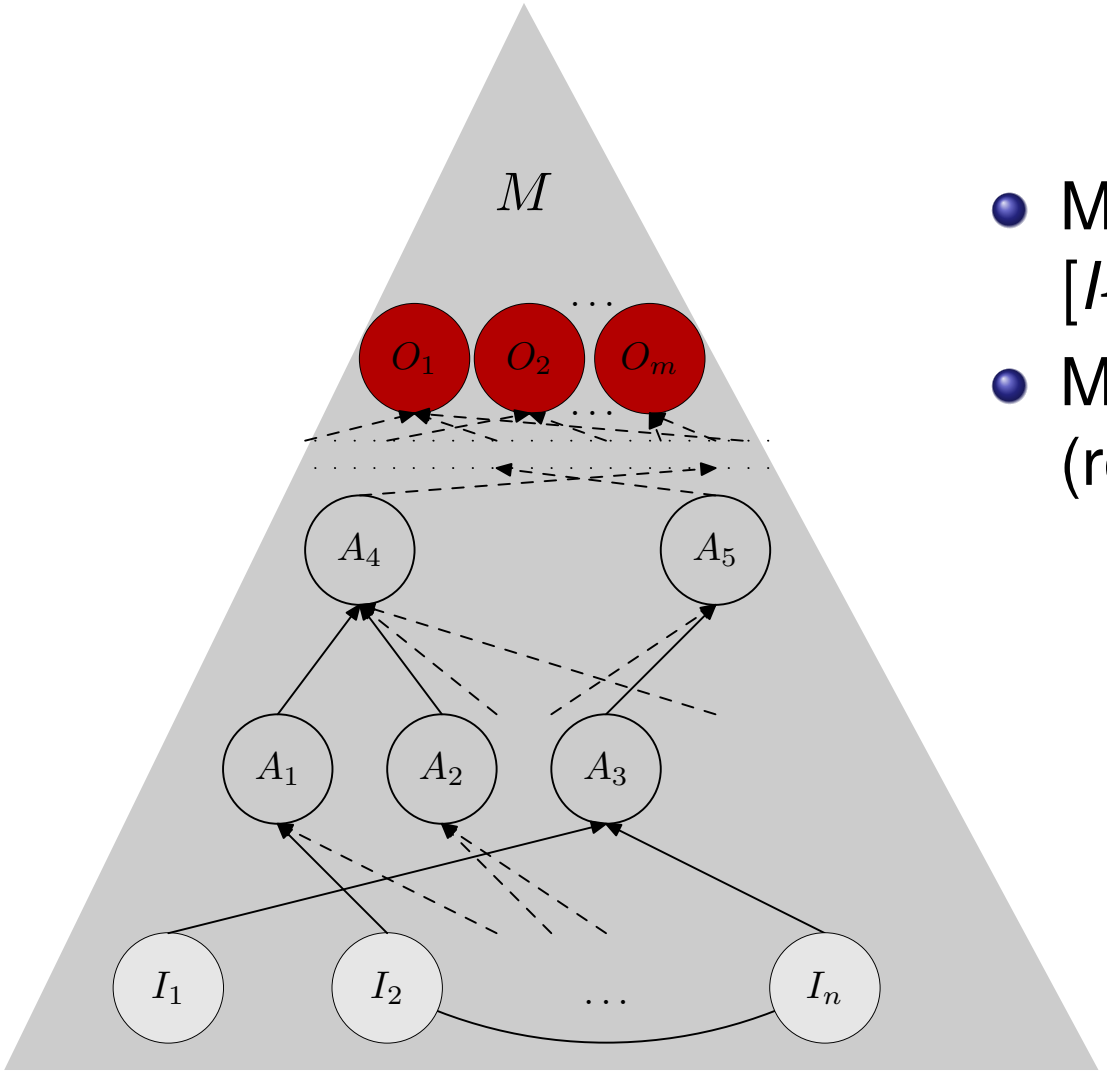
Model



- Model input attributes $[I_1, I_2, \ldots, I_n]$

Formalization - DEX

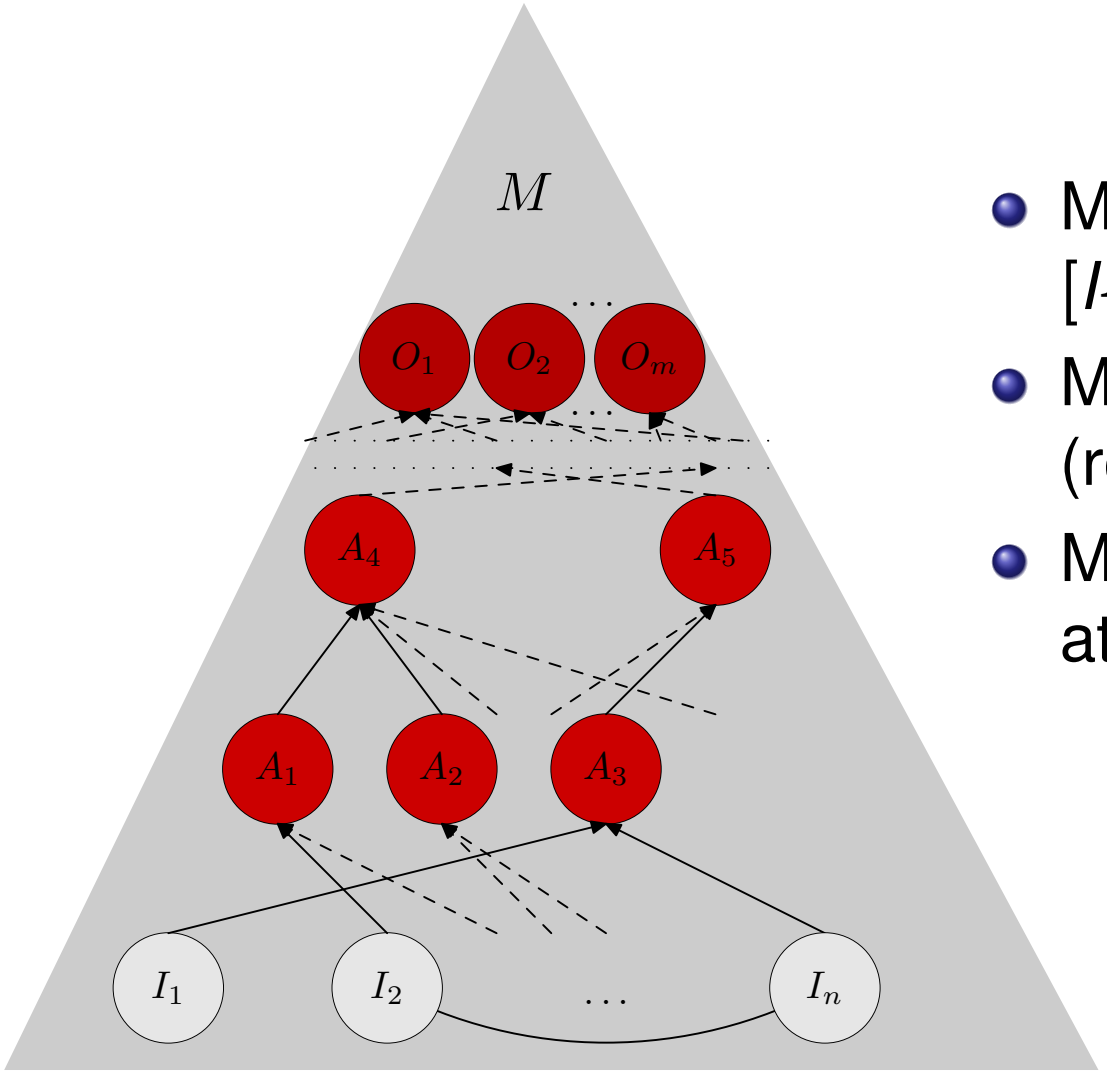
Model



- Model input attributes $[I_1, I_2, \ldots, I_n]$
- Model output attributes (roots) $[O_1, O_2, \ldots, O_m]$

Formalization - DEX

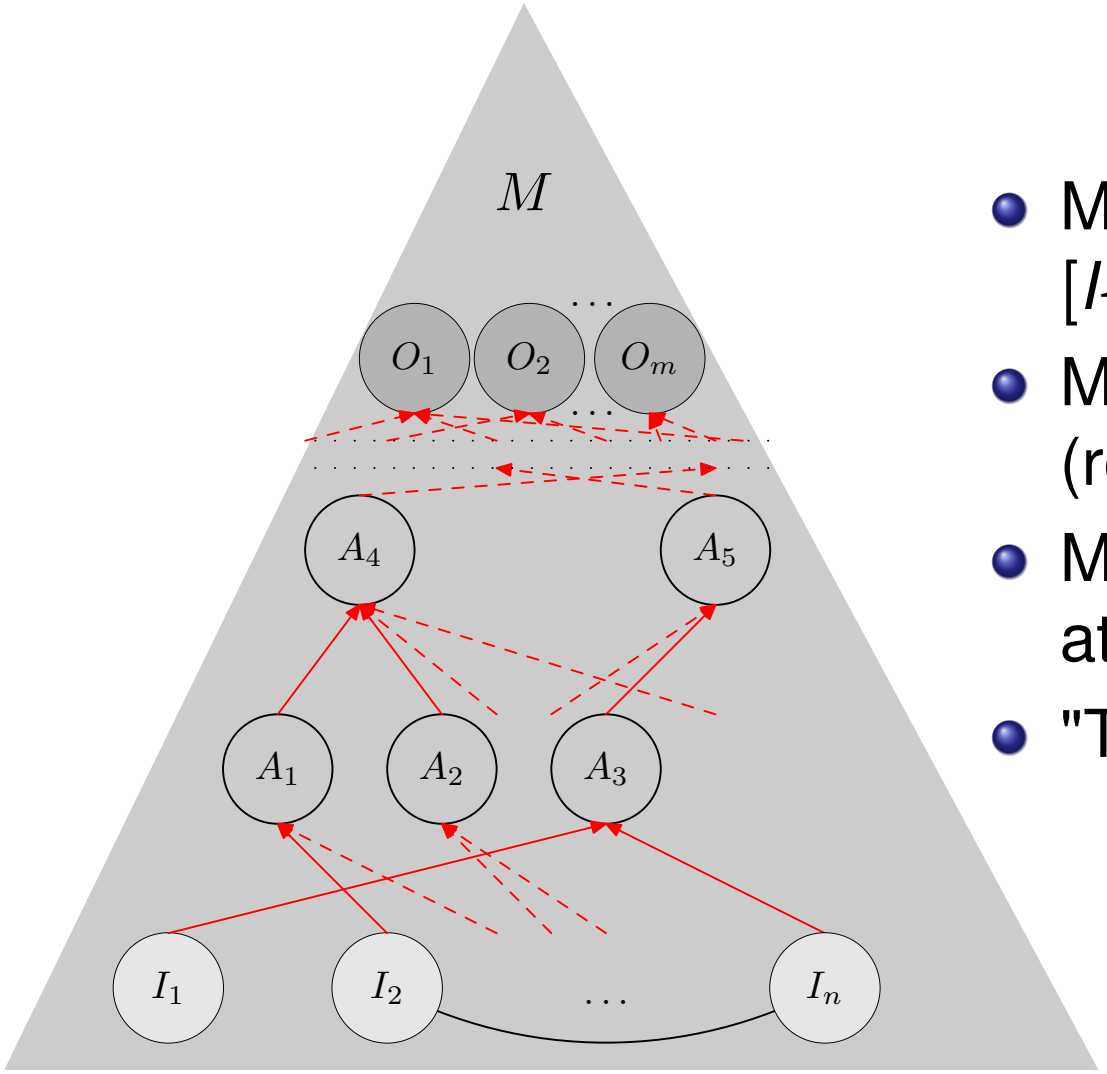
Model



- Model input attributes $[I_1, I_2, \ldots, I_n]$
- Model output attributes (roots) $[O_1, O_2, \ldots, O_m]$
- Model aggregated attributes $[A_1, A_2, \ldots, A_k]$

Formalization - DEX

Model



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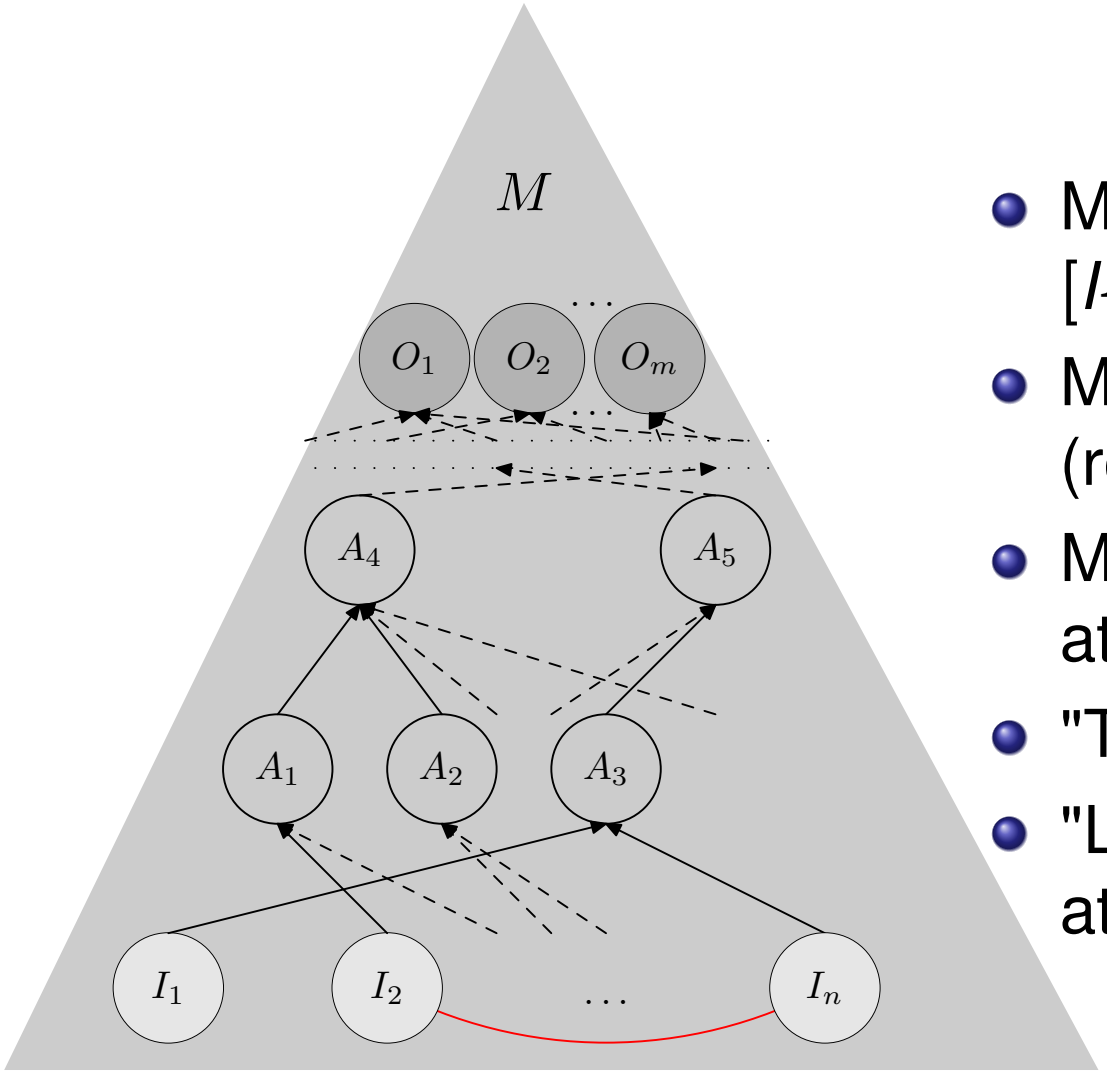
- Model aggregated attributes $[A_1, A_2, \ldots, A_k]$
- "Tree" of attributes

Formalization - DEX

Model



- Model input attributes $[I_1, I_2, \ldots, I_n]$
- Model output attributes (roots) $[O_1, O_2, \ldots, O_m]$
- Model aggregated attributes $[A_1, A_2, \ldots, A_k]$
- "Tree" of attributes
- "Links" between attributes

Qualitative relational models
Motivational example

Evaluation of a company

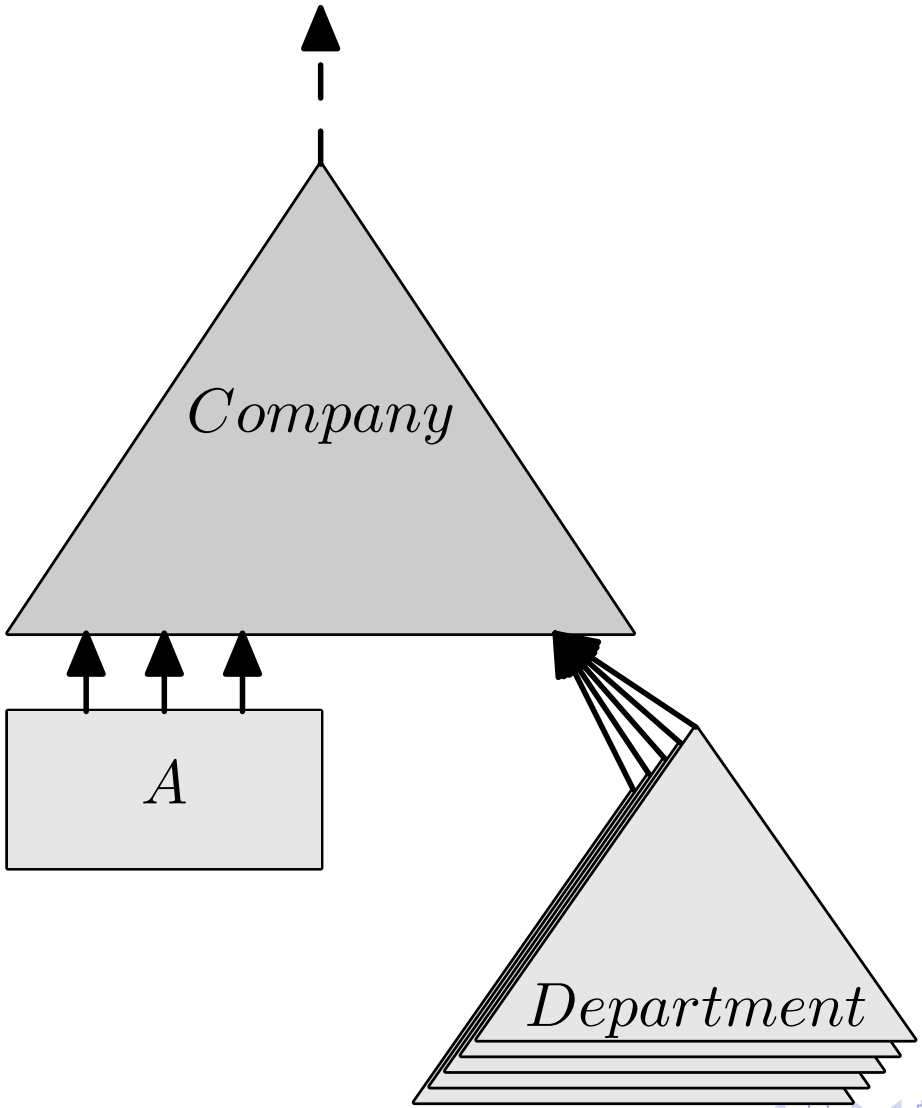
- Company consists of many departments
- Each department has to be evaluated. Each evaluation has to contribute to the final evaluation of a company.
- A company also has other "non-relational" attributes: Stock price, credit standing, number of buildings, number of employees, etc.

What is needed

- One model for evaluation of n departments
- Aggregation of n values into one single value
- One model for evaluation of the whole company

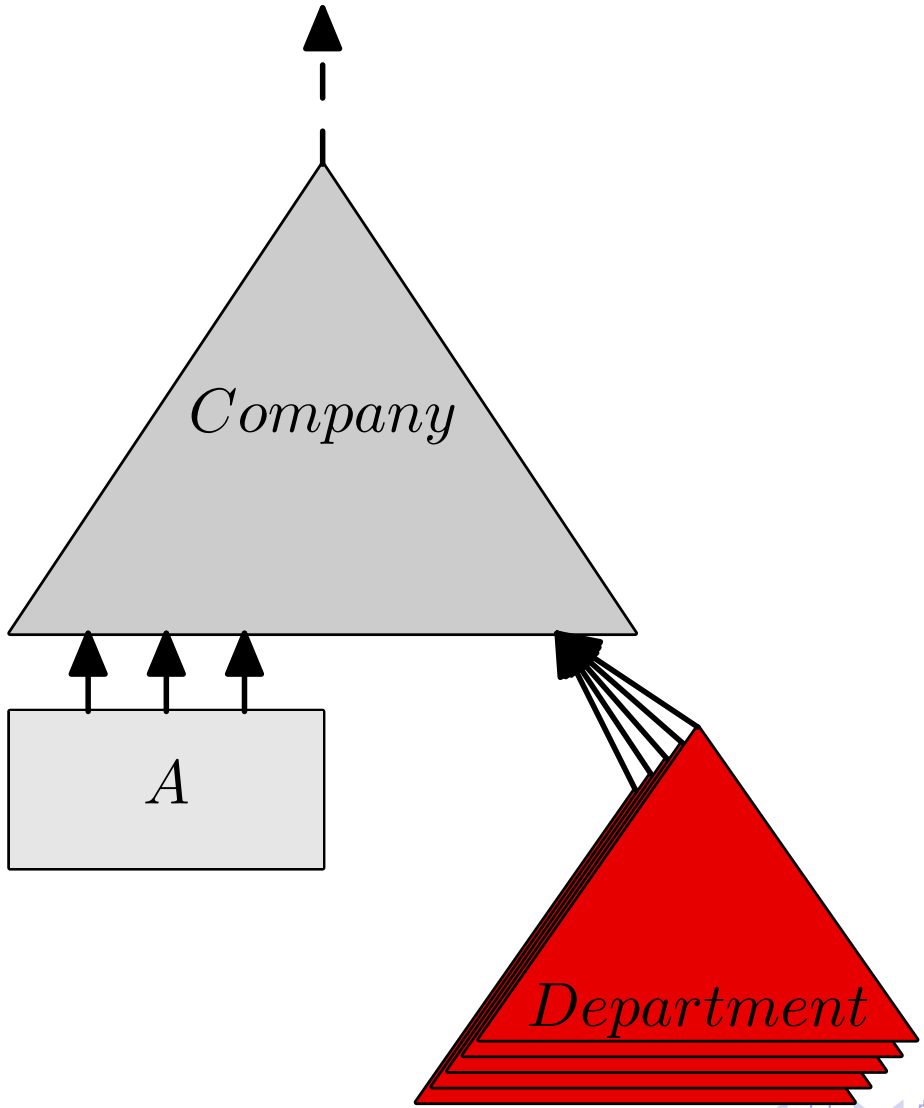
Qualitative relational models

Motivational example



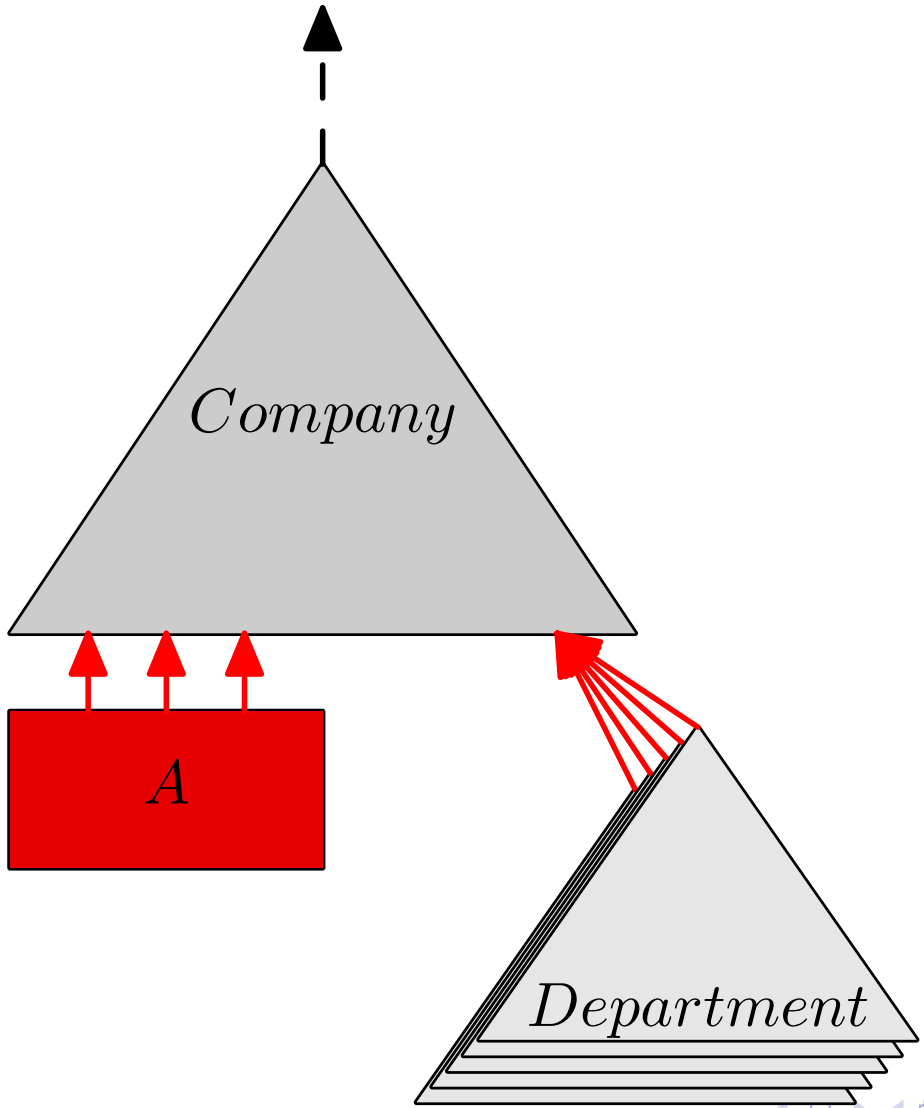
Qualitative relational models

Motivational example



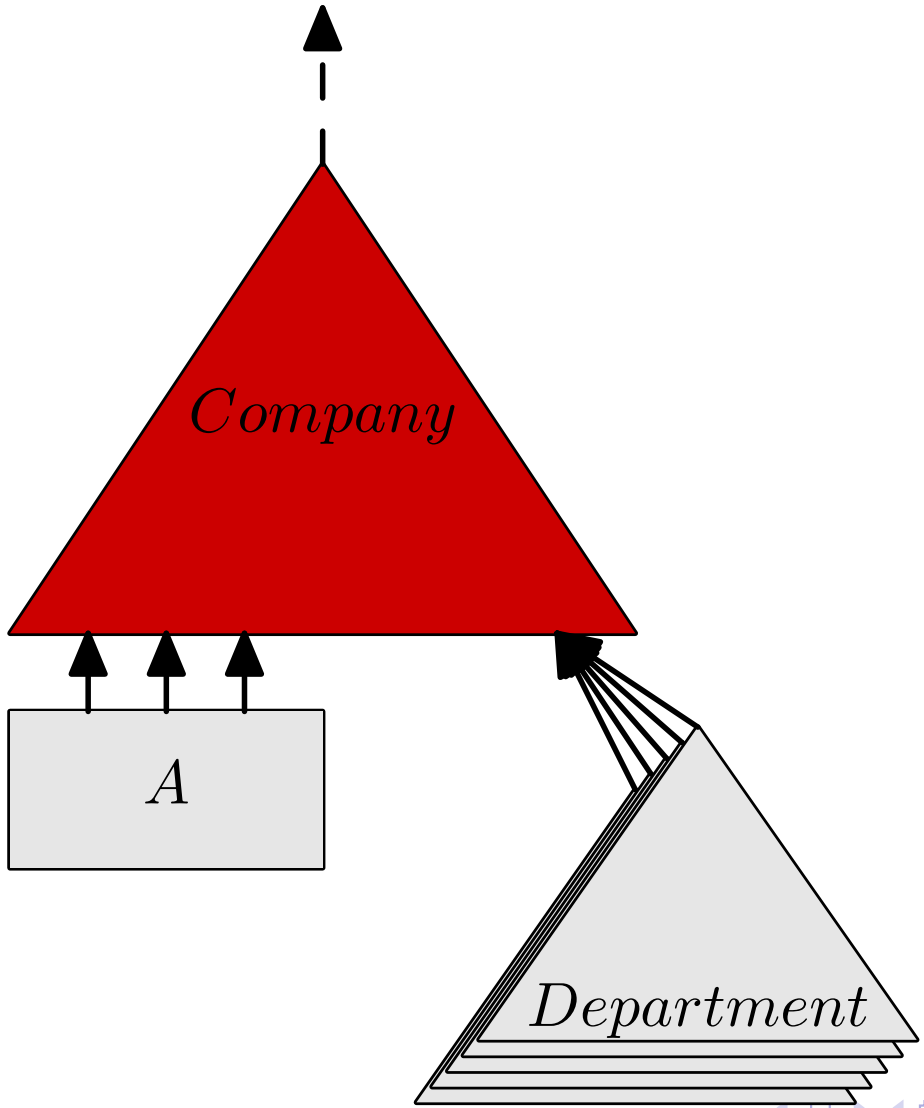
Qualitative relational models

Motivational example



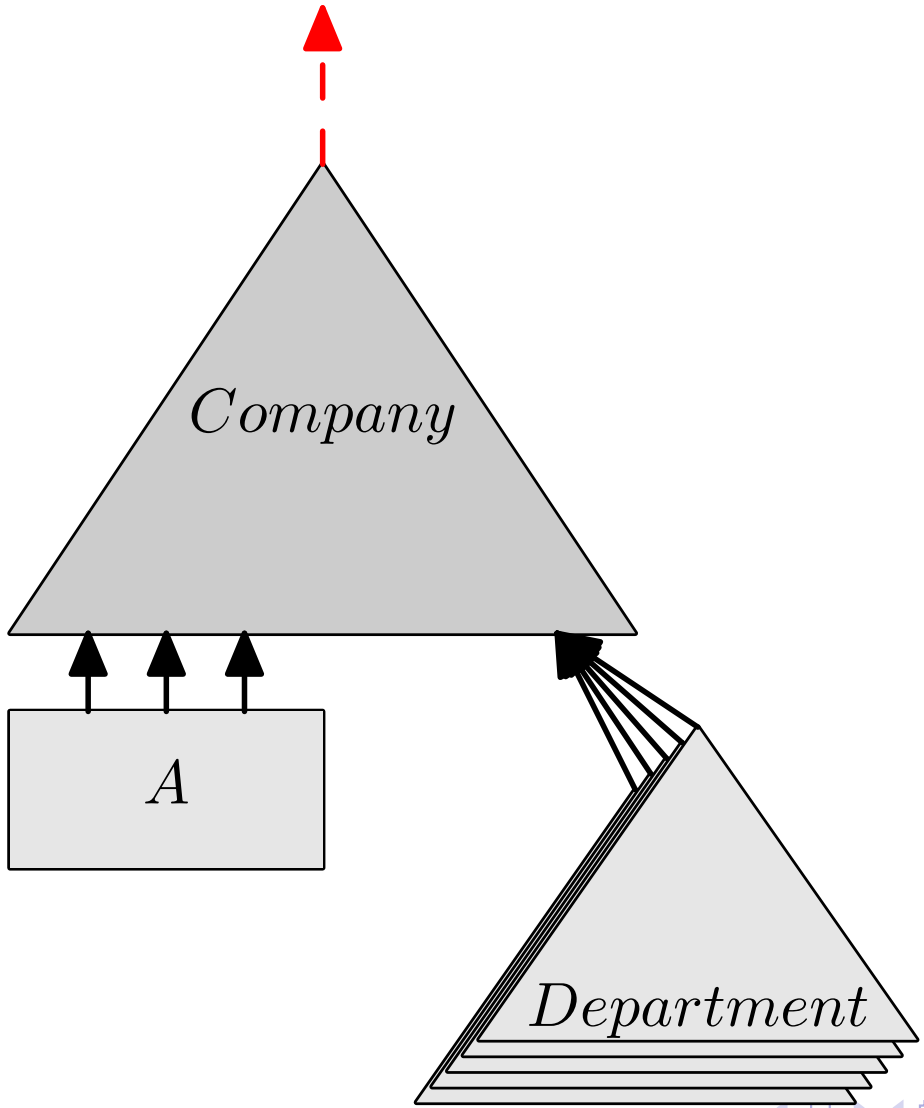
Qualitative relational models

Motivational example



Qualitative relational models

Motivational example



Qualitative relational models

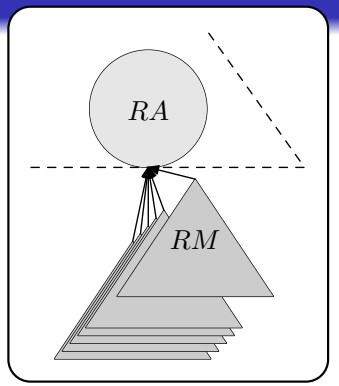
Definition

A decision problem/alternative has the *Relational property* when it is composed of arbitrary number of similar sub-components.

- Many decision problems have relational properties.
- For example:
  - Evaluation of a company;
  - Assessing reputational risk of a bank.
- In DEX, relational problems were so far solved by ad-hoc manual or programmatic manipulation of models.
- We propose an extension to DEX methodology.

Qualitative relational models

Relational aggregated attribute



- Relational aggregated attribute *RA*:
  - a special type of attribute.
  - it is placed as a model input.
  - has an input from arbitrary number ($m > 0$) of aggregated attributes belonging to some other relational model.
  - has one output.
  - Function
    $F_{RA} : (D(I_{RA,1}) \times D(I_{RA,2}) \times \ldots \times D(I_{RA,m}))^n \rightarrow D(RA)$.
- When $m = 1$, F_{RA} is typically a mathematical function: min, max, *mode* or *median*. We can compute a distribution, set or interval over n values.
- In general case, a custom function needs to be defined.

Qualitative relational models

- The qualitative relational model extension has been implemented and along with a qualitative use-case presented at a conference.

1. Trdin, N., Bohanec, M.: Relational multi-attribute models in DEX methodology. V: 22nd International Conference on Multiple Criteria Decision Making, 17-21 June 2013, Malaga, Spain. MCDM Malaga 2013. [S. l.: s. n.], 2013.

Numeric attributes

- More and more decision problems include qualitative and quantitative attributes.
- There is a need to aggregate such attributes.

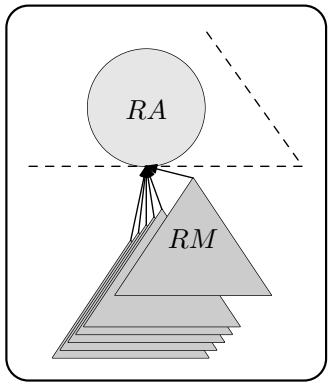
- 6 "basic" function types!
- The most problematic functions are the ones with qualitative and quantitative input attributes.

Addition

The domain of each attribute D, can now also be of numeric type (integer or real). The domain type is determined by the decision maker.

Numeric relational models

Numeric relational aggregated attribute



- Numeric relational aggregated attribute *RA*:
  - Function
    $F_{RA} : (D(I_{RA,1}) \times D(I_{RA,2}) \times \ldots \times D(I_{RA,m}))^n \rightarrow D(RA)$.
- When $m = 1$, F_{RA} is typically a mathematical function. Additionally to min, max, *mode* and *median*, we can compute *mean*. A distribution, set or an interval over n values can also be computed.
- In general case, a custom function of n numeric variables needs to be defined.

OVJE

Sustainability assessment of electric energy production technologies

- Sustainability assessment of electric energy production technologies in Slovenia with emphasis on nuclear technology.
- National project with an industrial partner.

1. Kontić, B., Kontić, D., Zagorc, S., Marušić, M., Dermol, U., Bohanec, M., Trdin, N.: Ocena vzdržnosti za razvoj energetike v Sloveniji do leta 2030 s poudarkom na jedrski tehnologiji. Book 1, (JSI work report, 11583). 2014.

OVJE
Modeling

- Two models:
  1. For evaluation of electricity production technologies.
  2. For evaluating a mix of electricity production technologies.
- Model (1): Evaluation of particular technology in general.
- Model (2): Evaluation of a technology mix, provided a mix of 8 technologies.
- Model (2) was also used for evaluation considering 6 events, from now till year 2050.
- Several attributes from model (1) were relationally connected to model (2).

OVJE

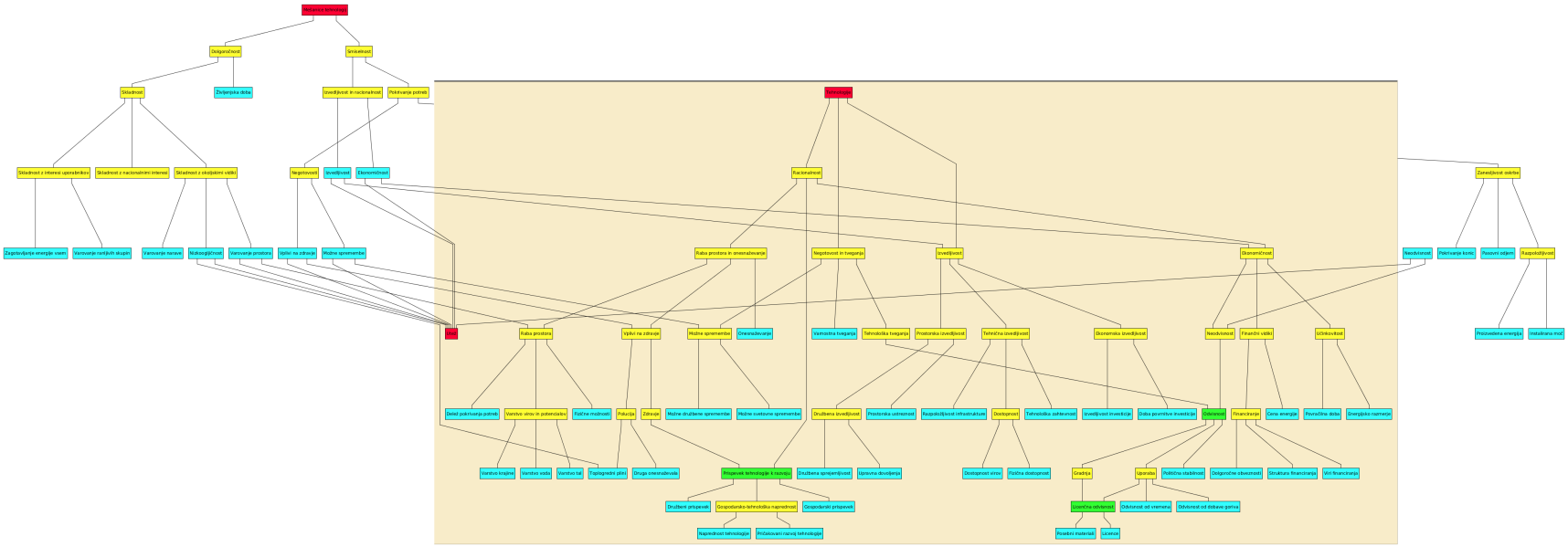
Overview of features used

- Mostly qualitative attributes and qualitative rule-based functions are used.
- Three numeric attributes are used for computing the weight of particular technology in the mix.
- Relational aggregation for assessing the evaluation of mix (used as input to model (2)).
- Distributions are used as inputs and also during evaluation.

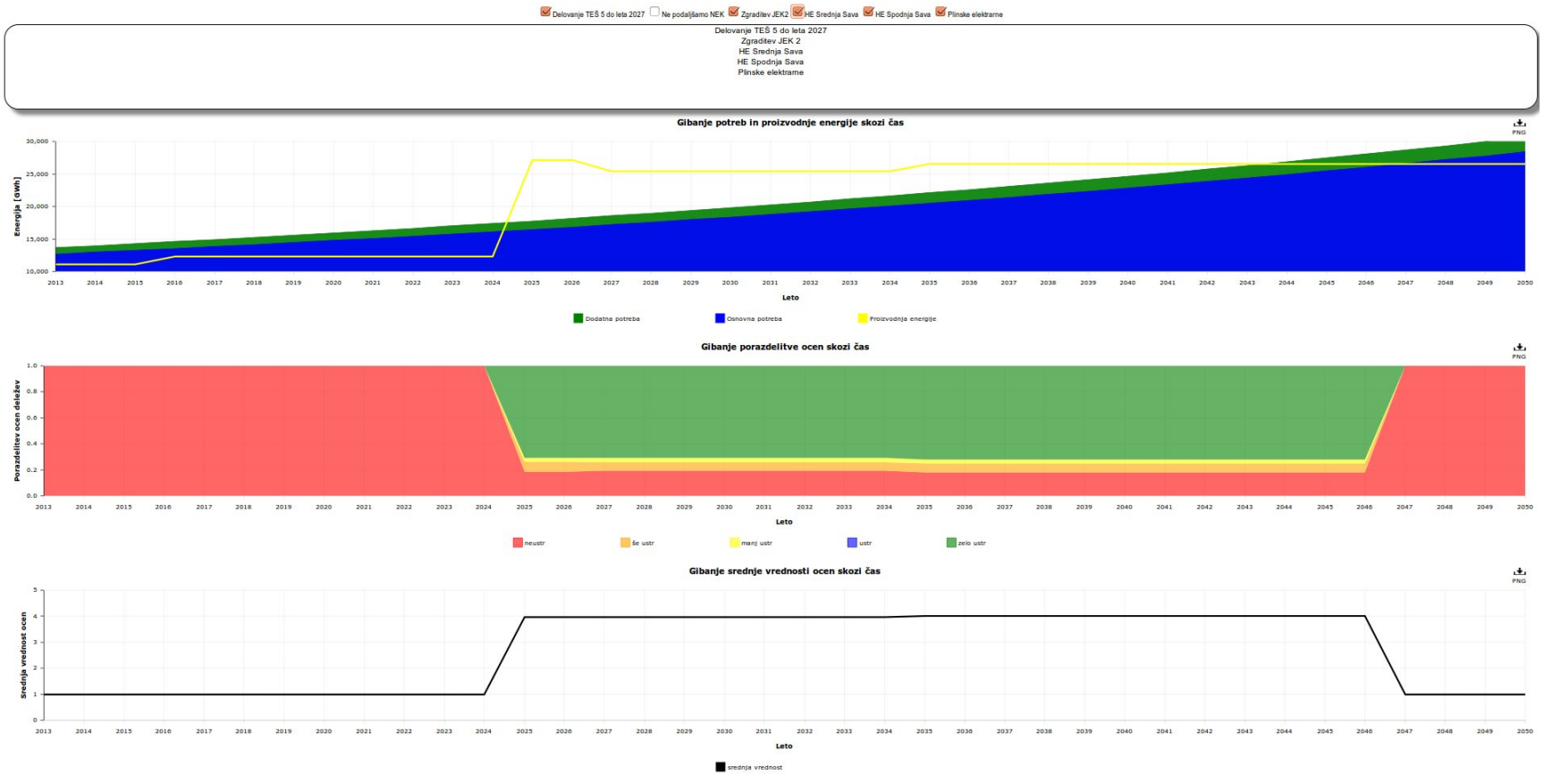
OVJE
Relational aggregated attributes

- 8 relationally aggregated attributes.
- Each has two inputs: (1) numeric and (2) qualitative:
  1. Contribution (weight) of each technology in the mix, based on the *power output* and *produced energy*.
  2. Certain important attributes from model (1): feasibility, health effects, carbon emission, etc.
- Aggregation is done by weighting each evaluation (from model (1)) with its corresponding weight. Generally, a distribution is output.

OVJE
Relational model



OVJE
Online evaluation view



Available on: `http://nejctrdin.com/ovjeGEN/`

Conclusion

- DEX methodology
- Qualitative and numeric relational models
- Use-case for utilization of numeric relational models