Numeric Relational Multi-Attribute Models in Qualitative Multi-Attribute Method DEX

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Motivation

- DEX is a qualitative MCDM methodology, supported by implementation in DEXi
- It has been applied numerous times; ecology, financial domains, evaluation of projects, medicine, etc.
- More flexibility in methodology is needed, specifically in the direction of relational models and numerical attributes.
Overview

1. DEX methodology
2. Formalization of DEX
3. Numeric relational models
4. Use-Case
5. Conclusion
The DEX methodology

State of the art: General

A qualitative multi-criteria decision making methodology
- Developed model is a hierarchy of attributes
- All attributes have symbolic values

Attribute $A$:
- most basic building block in the methodology.
- $D(A)$ is the domain: finite array of qualitative-symbolic values.

For example:
- $D(A_1) = [low, medium, high]$
- $D(A_2) = [yes, no]$
- $D(A_3) = [2\_or\_less, 3, 4, more\_than\_4]$
Aggregated attribute $A$:
- logical combination of lower level attributes...
- ..., into a higher level concept.

Total aggregation function (defined by a table)

$$F_A : D(A_1) \times D(A_2) \times \ldots \times D(A_n) \rightarrow I(D(A)).$$
Formalization - DEX
Aggregated attribute - Function example
Formalization - DEX

Model

M

I_1, I_2, ..., I_n

A_1, A_2, A_3, A_4, A_5

O_1, O_2, O_m

"Tree" of attributes

"Links" between attributes

Model input attributes

[ I_1, I_2, ..., I_n ]

Model output attributes (roots)

[ O_1, O_2, ..., O_m ]

Model aggregated attributes

[ A_1, A_2, ..., A_k ]
Formalization - DEX

Model

Model input attributes

\[ [I_1, I_2, \ldots, I_n] \]
Formalization - DEX

Model

- Model input attributes 
  \([I_1, I_2, \ldots, I_n]\)
- Model output attributes (roots) 
  \([O_1, O_2, \ldots, O_m]\)
Formalization - DEX

Model

- Model input attributes \([I_1, I_2, \ldots, I_n]\)
- Model output attributes (roots) \([O_1, O_2, \ldots, O_m]\)
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Formalization - DEX

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Formalization - DEX

Model

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- "Tree" of attributes
- "Links" between attributes
Qualitative relational models

Motivational example

Evaluation of a company

- Company consists of many departments
- Each department has to be evaluated. Each evaluation has to contribute to the final evaluation of a company.
- A company also has other "non-relational" attributes: Stock price, credit standing, number of buildings, number of employees, etc.

What is needed

- One model for evaluation of $n$ departments
- Aggregation of $n$ values into one single value
- One model for evaluation of the whole company
Qualitative relational models
Motivational example
Qualitative relational models
Motivational example
Qualitative relational models
Motivational example
Qualitative relational models
Motivational example

Company

A

Department
Qualitative relational models

Motivational example
# Qualitative relational models

## Definition

A decision problem/alternative has the *Relational property* when it is composed of arbitrary number of similar sub-components.

## Examples

- Many decision problems have relational properties.
- For example:
  - Evaluation of a company;
  - Assessing reputational risk of a bank.
- In DEX, relational problems were so far solved by ad-hoc manual or programmatic manipulation of models.
- We propose an extension to DEX methodology.
Relational aggregated attribute $RA$:
- a special type of attribute.
- it is placed as a model input.
- has an input from arbitrary number ($m > 0$) of aggregated attributes belonging to some other relational model.
- has one output.
- Function
  \[
  F_{RA} : (D(I_{RA,1}) \times D(I_{RA,2}) \times \ldots \times D(I_{RA,m}))^n \rightarrow D(RA).
  \]
- When $m = 1$, $F_{RA}$ is typically a mathematical function: min, max, mode or median. We can compute a distribution, set or interval over $n$ values.
- In general case, a custom function needs to be defined.
Qualitative relational models

The qualitative relational model extension has been implemented and along with a qualitative use-case presented at a conference.

Numeric attributes

- More and more decision problems include qualitative and quantitative attributes.
- There is a need to aggregate such attributes.

- 6 "basic" function types!
- The most problematic functions are the ones with qualitative and quantitative input attributes.

Addition

The domain of each attribute $D$, can now also be of numeric type (integer or real). The domain type is determined by the decision maker.
Numeric relational models

Numeric relational aggregated attribute

- Numeric relational aggregated attribute $RA$:
  - Function
    \[ F_{RA} : (D(I_{RA,1}) \times D(I_{RA,2}) \times \ldots \times D(I_{RA,m}))^n \rightarrow D(RA). \]
  - When $m = 1$, $F_{RA}$ is typically a mathematical function. Additionally to min, max, mode and median, we can compute mean. A distribution, set or an interval over $n$ values can also be computed.
  - In general case, a custom function of $n$ numeric variables needs to be defined.
OVJE
Sustainability assessment of electric energy production technologies

- Sustainability assessment of electric energy production technologies in Slovenia with emphasis on nuclear technology.
- National project with an industrial partner.

Two models:
1. For evaluation of electricity production technologies.
2. For evaluating a mix of electricity production technologies.

Model (1): Evaluation of particular technology in general.

Model (2): Evaluation of a technology mix, provided a mix of 8 technologies.

Model (2) was also used for evaluation considering 6 events, from now till year 2050.

Several attributes from model (1) were relationally connected to model (2).
OVJE
Overview of features used

- Mostly qualitative attributes and qualitative rule-based functions are used.
- Three numeric attributes are used for computing the weight of particular technology in the mix.
- Relational aggregation for assessing the evaluation of mix (used as input to model (2)).
- Distributions are used as inputs and also during evaluation.
8 relationally aggregated attributes.

Each has two inputs: (1) numeric and (2) qualitative:
1. Contribution (weight) of each technology in the mix, based on the *power output* and *produced energy*.
2. Certain important attributes from model (1): feasibility, health effects, carbon emission, etc.

Aggregation is done by weighting each evaluation (from model (1)) with its corresponding weight. Generally, a distribution is output.
OVJE

Online evaluation view

Available on: http://nejctrdin.com/ovjeGEN/
Conclusion

- DEX methodology
- Qualitative and numeric relational models
- Use-case for utilization of numeric relational models