Decision Support: Decision Analysis

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Programme: Information and Communication Technologies (ICT3)
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Decision Analysis

Part 1
Decision Analysis and Decision Tables

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Decision Analysis, Part 1

- Introduction to Decision Analysis
  - Concepts: modeling, evaluation, analysis
  - Decision Problem-Solving: Stages
  - Relation of DA to some other Disciplines
- Decision-Making under Uncertainty
  - Decision-Making under Strict Uncertainty
    - Decision Table
    - Various Decision Criteria
  - Decision-Making under Risk
    - Expected Value
    - Sensitivity Analysis

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Decision Analysis

Decision Analysis: Applied Decision Theory

Provides a framework for analyzing decision problems by
- structuring and breaking them down into more manageable parts,
- explicitly considering the:
  - possible alternatives,
  - available information
  - uncertainties involved, and
  - relevant preferences
- combining these to arrive at optimal (or "good") decisions

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Decision-Making Problem

- options (alternatives)
- goals (objectives)
  - FIND the option that best satisfies the goals
  - RANK options according to the goals
  - ANALYSE, JUSTIFY, EXPLAIN, …, the decision

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Evaluation Models

- options
  - EVALUATION
  - MODEL
  - ANALYSIS

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Decision Support and Decision Modeling

Decision Support
Modelling
Decision Analysis

Decision-Making under Uncertainty

Decision-Making Problem
Suppose that one must choose between several uncertain alternatives.

Given:
- Alternatives;
- The consequences of choosing each alternative, described with a single number, e.g. profit / loss in € or aggregated value.

Task: Which alternative to choose?

Decision Table
Decision-Making under Strict Uncertainty
State of the world (Event) | Value of alternatives 1 ... m
---|---
\( \Theta_1 \) | \( a_1 \) ... \( a_m \)
\( \Theta_2 \) | \( y_{11} \) ... \( y_{1m} \)
| : | |
\( \Theta_n \) | \( y_{n1} \) ... \( y_{nm} \)

Decision-Making under Risk
State of the world (Event) | Probability that \( \Theta \) will happen | Value of alternatives 1 ... m
---|---|---
\( \Theta_1 \) | \( P(\Theta_1) \) | \( y_{11} \) ... \( y_{1m} \)
| : | : | : |
\( \Theta_n \) | \( P(\Theta_n) \) | \( y_{n1} \) ... \( y_{nm} \)

Working Example
A manufacturing company, faced with a possible increase in demand for its product, considers the following:

Alternatives:
1. status quo: no change
2. extend: extending their production line buying a new machine
3. build: building a new production hall with new equipment
4. cooperate: finding additional business partners for production

Uncertainty involved: Market reaction: after the decision, the sales can increase or decrease.
Consequences: Expected profit, shown in decision table on the next slide

Working Example
Decision table

<table>
<thead>
<tr>
<th>alternative</th>
<th>decreased sales</th>
<th>increased sales</th>
</tr>
</thead>
<tbody>
<tr>
<td>status quo</td>
<td>28</td>
<td>30</td>
</tr>
<tr>
<td>extend</td>
<td>24</td>
<td>42</td>
</tr>
<tr>
<td>build</td>
<td>16</td>
<td>44</td>
</tr>
<tr>
<td>cooperate</td>
<td>30</td>
<td>34</td>
</tr>
</tbody>
</table>
Decision-Making under Strict Uncertainty

Decision Criteria
- Dominance
- Pessimistic (Maximin, Wald’s)
- Optimistic (Maximax)
- Hurwicz’s
- Laplace’s
- Minimax Regret

Dominance
- Choose the alternative with best consequences in all states of the world.
- Such alternative is seldom found.

<table>
<thead>
<tr>
<th>alternative</th>
<th>status quo</th>
<th>extend</th>
<th>build</th>
<th>cooperate</th>
</tr>
</thead>
<tbody>
<tr>
<td>decreased sales</td>
<td>28</td>
<td>24</td>
<td>16</td>
<td>30</td>
</tr>
<tr>
<td>increased sales</td>
<td>30</td>
<td>42</td>
<td>44</td>
<td>34</td>
</tr>
</tbody>
</table>

No dominant alternatives in this case

Pessimistic Criterion (Wald’s, Maximin)
- Each alternative is represented by its worst possible consequence.
- According to these, the alternative with the best worst case is chosen.

<table>
<thead>
<tr>
<th>alternative</th>
<th>status quo</th>
<th>extend</th>
<th>build</th>
<th>cooperate</th>
</tr>
</thead>
<tbody>
<tr>
<td>decreased sales</td>
<td>28</td>
<td>24</td>
<td>16</td>
<td>30</td>
</tr>
<tr>
<td>increased sales</td>
<td>30</td>
<td>42</td>
<td>44</td>
<td>34</td>
</tr>
</tbody>
</table>

Pessimist: 28, 24, 16, 30

Optimistic Criterion (Maximax)
- Each alternative is represented by its best possible consequence.
- The alternative for which this best consequence is best is chosen.

<table>
<thead>
<tr>
<th>alternative</th>
<th>status quo</th>
<th>extend</th>
<th>build</th>
<th>cooperate</th>
</tr>
</thead>
<tbody>
<tr>
<td>decreased sales</td>
<td>28</td>
<td>24</td>
<td>16</td>
<td>30</td>
</tr>
<tr>
<td>increased sales</td>
<td>30</td>
<td>42</td>
<td>44</td>
<td>34</td>
</tr>
</tbody>
</table>

Optimist: 30, 42, 44, 34

Hurwicz’s Criterion
- Introduce a parameter $d \in [0,1]$. Combine Optimistic and Pessimistic criteria so that $u_i = du_p + (1-d)u_o$.

<table>
<thead>
<tr>
<th>alternative</th>
<th>status quo</th>
<th>extend</th>
<th>build</th>
<th>cooperate</th>
</tr>
</thead>
<tbody>
<tr>
<td>decreased sales</td>
<td>28</td>
<td>24</td>
<td>16</td>
<td>30</td>
</tr>
<tr>
<td>increased sales</td>
<td>30</td>
<td>42</td>
<td>44</td>
<td>34</td>
</tr>
</tbody>
</table>

Pessimist: 28, 24, 16, 30

Optimist: 30, 42, 44, 34

Hurwicz ($d=0.3$): 28.6, 29.4, 24.4, 21.3
Hurwicz’s Criterion

The regret $r_j$ for the alternative $a_j$ in state $\theta_i$ is equal to the difference between the best alternative in given state $\theta_i$ and $a_j$: $r_j = \max_i \{y_i\} - y_j$.

Choose the alternative having the least maximum regret.

Laplace’s Criterion

• Consider all states (events) equally likely.
• thus, consider the average of outcomes for each alternative.

Minimax Regret

Choose the alternative having the least maximum regret.

Summary

• What do you think about the criteria:
  - Are they comprehensible?
  - Are they realistic?
  - Are they useful for practice?
  - Which is your favourite criterion?
  - Is there a single “best” criterion? Which and why?

Questions

• If you were the manager, which alternative would you take? Why?
• Is this really the best alternative? Why? Under which circumstances it is best?
• What can you say about the status quo alternative? According to the analysis, when should be it taken, or should it be taken at all?
Decision-Making under Risk

**Working Example**

Now we know (or estimate) the probability of states

<table>
<thead>
<tr>
<th>states</th>
<th>probability</th>
<th>alternatives</th>
<th>status quo</th>
<th>extend</th>
<th>build</th>
<th>cooperate</th>
</tr>
</thead>
<tbody>
<tr>
<td>decreased sales</td>
<td>25%</td>
<td>28</td>
<td>24</td>
<td>16</td>
<td>30</td>
<td></td>
</tr>
<tr>
<td>increased sales</td>
<td>75%</td>
<td>30</td>
<td>42</td>
<td>44</td>
<td>34</td>
<td></td>
</tr>
</tbody>
</table>

**Decision Criteria**

- Mode: Select the most probable state
- Expected Value (EV), Expected Monetary Value (EMV)

**Expected (Monetary) Value**

Maximise the expected value:

\[ EV = \sum_{j=1}^{n} p(\theta_j)v_j \]

<table>
<thead>
<tr>
<th>states</th>
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<th>alternatives</th>
<th>status quo</th>
<th>extend</th>
<th>build</th>
<th>cooperate</th>
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<tbody>
<tr>
<td>decreased sales</td>
<td>25%</td>
<td>28</td>
<td>24</td>
<td>16</td>
<td>30</td>
<td></td>
</tr>
<tr>
<td>increased sales</td>
<td>75%</td>
<td>30</td>
<td>42</td>
<td>44</td>
<td>34</td>
<td></td>
</tr>
</tbody>
</table>

**Expected value**

\[
0.25 \times 28 + 0.75 \times 30 = 29.5 \\
0.25 \times 24 + 0.75 \times 42 = 37.5 \\
0.25 \times 16 + 0.75 \times 44 = 37 \\
0.25 \times 30 + 0.75 \times 34 = 33
\]

**Exercise 1**

Given this decision table:

- Determine which alternative is best according to all the criteria (Dominance, Pessimistic, Optimistic, Hurwiz \(d=0.7\), Laplace, Regret, Mode, Expected Value).
- Draw a chart evaluating the Hurwiz’s criterion for \(d \in [0,1]\).
- Do sensitivity analysis.
Exercise 2

Help the farmer who is deciding which crop to plant in the face of uncertain weather and resulting crop yield:

<table>
<thead>
<tr>
<th>probability</th>
<th>Weather</th>
<th>Normal</th>
<th>.15</th>
<th>30</th>
</tr>
</thead>
<tbody>
<tr>
<td>Plant soybeans</td>
<td>$10</td>
<td>5</td>
<td>12</td>
<td></td>
</tr>
<tr>
<td>Plant corn</td>
<td>7</td>
<td>8</td>
<td>13</td>
<td></td>
</tr>
</tbody>
</table>

profit per acre

Exercise 3

1. Define a decision problem of your own,
2. represent it in a decision table,
3. and repeat the steps of Exercise 1

Exercise 4

Using some decision table, implement in spreadsheet software (such as MS Excel):

- evaluation of alternatives using all the criteria,
- drawing the chart associated with Hurwicz’s criterion
- drawing the sensitivity analysis chart

Compare the two charts.

Decision Analysis

Part 2: Decision Trees

Working Example

Decision table (Payoff matrix)

<table>
<thead>
<tr>
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<td>42</td>
<td>44</td>
<td>34</td>
</tr>
</tbody>
</table>

Working Example

Equivalent decision tree:

- alternatives
  - status quo
  - extend
  - build
  - cooperate
- events (states)
  - decreased sales (0.25)
  - increased sales (0.75)
- expected profit
  - 28
  - 30
  - 24
  - 42
  - 16
  - 44
  - 30
  - 34
**Decision Tree**

Different from decision trees used in Machine Learning:
- different types of nodes
- always drawn horizontally, from left to right
- “hand-crafted”, not learned from data

Decision tree represents the decision problem in terms of chains of consecutive decisions and chance events. 
*Time proceeds from left to right.*

Uncertainties associated with chance events are modelled by probabilities.

**Decision Tree Development**

1. Place decision and chance nodes in a logical time order
2. Independent chance nodes can be placed in any order
3. Estimate probabilities of all chance events
4. The sum of probabilities in a chance node must be 1
5. In terminal nodes, specify consequences by a single performance measure, e.g.:
   - money,
   - aggregate utility or
   - results of a multiple criteria analysis

**Components of Decision Trees**

- **Decision node:** represents alternatives
- **Chance Node:** represents events (states of nature)
- **Terminal (End) Node:** represents consequences of decisions

**Solving Decision Trees**

From right to left:

- \( E'V = \max_i E'V_i \) [maximize profit]
- or \( E'V = \min_i E'V_i \) [minimise losses]

\[ E'V = \sum p_i E'V_i \]

**Solved Decision Tree**

- **Expected profit**
- **Proportion**

**Common Mistakes**

1. Decision and chance nodes are in wrong order:
   - Only chance nodes whose results are known at the time of decision can precede a decision node
2. Incorrect derivation of chance probabilities:
   - Chance probabilities depend on each other and decisions made
3. Chance events with probability 0 can be left out
4. When solving the tree:
   - Maximising instead of minimising, or vice versa
Example 1: Oilco

Mobon Oil Company has a lease on an offshore oil site. The lease is about to expire and they are faced with either developing the field or selling the lease to Excel Oil Co. for $50,000. It costs approximately $100,000 to drill a well. There is a 45% chance that the well is dry, a 45% chance that the well will have a minor strike and a 10% chance that they will have a major strike. For a typical minor strike the revenues average $300,000. If the strike is major the revenues average $700,000. What should Mobon do?

Example 2: Sequential Decision

Other Important Concepts

1. Value of Perfect Information
2. Risk Profile

Example

- Oilco must determine whether or not to drill for oil in the South China Sea. It costs $100,000 to drill for oil and if oil is found the value of the oil is estimated to be $600,000. At present, Oilco believes there is a 45% chance that the field contains oil. What should Oilco do?
- What is the value of perfect information (knowledge of whether the field contains oil) to Oilco?
Ordinary Decision Tree

Recommendation: Drill!

$270K - $100K = $170K
0.55*0 = 0
0.45*600K = 270K
Total = $270K

Value of Perfect Information

Exchange decision and event nodes:

Value of Perfect Information: $225 - $170 = $55

Solved Decision Tree

Risk Profile

Alternative: extend

Cumulative Risk Profile

All alternatives
Decision Tree Software

Add-Ins for Microsoft Excel:
- TreePlan: http://www.treeplan.com/
- PrecisionTree: http://www.palisade-europe.com/precisiontree/

Decision-Tree Development Programs:
- TreeAge Pro (DATA): http://www.treeage.com/
- DecisionPro: http://www.vanguardsw.com/
- DPL: http://www.syncopationsoftware.com/

Decision Analysis Software

http://decision-analysis.society.informs.org/Field/FieldSoftware.html

See also: OR/MS Today, December 2006
http://www.lionhrtpub.com/orms/surveys/das/das.html
A Typical Application in Medicine

A Cost-Benefit Analysis of Testing for Influenza A in High-Risk Adults

Authors: Smith et al.

Abstract

Influenza A viral disease and vaccine therapy have been strategies for treatment of patients infected with high-risk groups. This study used cost-effectiveness analysis to evaluate the cost of influenza A testing in a group of high-risk patients.

Methods

A decision tree was developed to assess the cost-effectiveness of testing for influenza A in high-risk patients. A base case scenario was created using data from a recent study. Sensitivity analysis was performed to test the robustness of the results.

Results

The decision tree showed that testing for influenza A in high-risk patients was cost-effective. The incremental cost-effectiveness ratio was below the threshold of $100,000 per quality-adjusted life year gained.

Conclusion

Testing for influenza A in high-risk patients is a cost-effective strategy.

Questions

1. What do decision trees facilitate that decision tables don’t?
2. Identify limitations and/or shortcomings of decision trees.
3. Identify types of decision problems suitable for the application of decision trees.

Exercise 1

Consider the decision tree:

1. Solve the tree for tomorrow’s p
2. Do sensitivity analysis
3. Take the risk profile of your decision

Exercise 2

For the decision tree shown in the slide “Example 2: Sequential Decision” (Introduce Product):

1. do sensitivity analysis with respect to p(competitor)
2. find the risk profile of alternative introduce.

Tree Development Exercise (1/3)

Service station problem:

- You are the owner of a service station on an intercity road. You have heard a rumour that the road may be upgraded or diverted along a different route. What do you do? What information will you need? How do you formulate a decision model?
- Think: before reading next slides, structure your own decision. You will need to specify an objective, identify alternatives available to you as the service station owner, and identify the uncertainties involved in this decision situation together with the possible events.
Tree Development Exercise (2/3)

Possible answers to the questions from the previous slide:

**Objective:** maximise the value of service station investment

**Alternatives:**
- status quo
- self
- extend

**Events:**
- unaltered ($p=0.5$)
- upgrade ($p=0.3$)
- divert ($p=0.2$)

Tree Development Exercise (3/3)

Proceed as follows:
1. Define decision table (include consequences)
2. Convert decision table to decision tree
3. Calculate **$E_{VP}$** and identify the best alternative
4. Do sensitivity analysis with respect to $p$ (unaltered)
   - which problem do you encounter here?
5. Find the risk profile of the best alternative

Decision Analysis

Part 3:

Influence Diagrams

Motivation for Influence Diagrams

Decision trees:
- sometimes too detailed,
- grow exponentially,
- contain repeated information

Only three different elements:

Working Example

Decision tree:

Equivalent Influence diagram:
Influence Diagram

**Influence diagram is a:**
- high-level (compact),
- visual representation,
- displaying relationships between essential elements that affect the decision.

Two levels of detail:
- higher: only elements and relations
- lower: detailed information defined with each element

Elements of Influence Diagrams

- **Decision node:** represents alternatives
- **Chance Node:** represents events (states of nature)
- **Value Node** represents:
  - consequences
  - objectives, or
  - calculations

Arcs in Influence Diagrams

- Decision A affects the probabilities of event B; Decision A is relevant for event B
- The outcome of event A affects the probabilities of event B; Event A is relevant for event B
- Decision A occurs before decision B; Decisions A and B are sequential
- Decision B occurs after event A; The outcome of A is known when deciding about B

Developing Influence Diagrams

Two basic strategies:
- Start with outcomes and model towards decisions and events
- Gradually add more and more detail

Common Mistakes

1. An influence diagram is not a flowchart.
2. An arc from a chance node into a decision node means that the decision-maker knows the outcome of the chance node when making the decision.
3. There can be no cycles:

Decision Trees : Influence Diagrams

- DT display more information, the details of a problem, but they may become "messy".
- ID show a general structure of a problem and hide details.
- ID are particularly valuable for the structuring phase of problem solving and for representing large problems.
- Solving algorithms: DT straightforward, ID difficult
- Any properly built ID can be converted into a DT, and vice versa.
  - Bayesian networks are ID's containing only event nodes
Solving Influence Diagrams

A. Convert ID to DT, solve DT
or
B. Solve directly by node reduction:
   1. Cleanup: one consequence $C'$, no cycles, transform calculation nodes to one-event chance nodes...
   2. Repeat until ID solved:
      1. Reduce (calculate $E[F]$) of all chance nodes that directly precede $C'$ and do not precede any other node.
      2. Reduce (calculate $E[F]$) of the decision node that directly precedes $C'$ and has as predecessors all of the other direct predecessors of $C'$.
   + arc reversal where there are no nodes corresponding to 2.2

Influence Diagram Software

Add-Ins for Microsoft Excel:

- PrecisionTree: http://www.palisade-europe.com/precisiontree/

Influence-Diagram Development Programs:

- GeNe: http://gene.sis.pitt.edu/
- TreeAge Pro (DATA): http://www.treeage.com/
- DPL: http://www.syncopationsoftware.com/
- Analytica: http://www.lumina.com/ana/whatisanalytica.htm
- HUGIN: http://www.hugin.com/
- Netica: http://www.norsys.com/

Exercises

Exercise 1: DATA

Exercise 2: GeNe

Exercise 3: Develop ID
Example 1: Gradual Development

Influence diagram of a new product decision

- Revenue
- Fixed Cost
- Units Sold
- Variable Cost
- Introduce Product?
- Price
- Profit
- Cost

Example 1: Gradual Development

Influence diagram with additional detail

- Price
- Units Sold
- Fixed Cost
- Variable Cost
- Introduce Product?
- Profit
- Cost
- Intermediate calculation

Example 2: Multiple Objectives

Influence diagram of a venture capitalist’s decision

- Venture succeeds or fails
- Invest?
- Return on investment
- Computer Industry Growth
- Pollution Level
- Number of Cars
- New Plant Licensed
- New Regulations
- Build New Plant?
- Plant Profit

Example 3: Intermediate Calculations
Example 4: Evacuation Decision

Exercise 5: Tractor Buying

Exercise 4
Create influence diagrams representing the decision trees encountered so far:

1. Oilco
2. Take an umbrella
3. Service station

Exercise 5: Tractor Buying

Exercise 5: Tractor Buying

Exercise 5: Tractor Buying