Decision Support: Decision Analysis

Jožef Stefan International Postgraduate School, Ljubljana
Programme: Information and Communication Technologies [ICT3]
Course Web Page: http://kt.ijs.si/MarkoBohanec/DS/DS.html

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Decision Analysis Part 1

Decision Analysis and Decision Tables

Decision Analysis, Part 1

- Introduction to Decision Analysis
  - Concepts: modelling, evaluation, analysis
  - Decision Problem-Solving: Stages
  - Relation of DA to some other Disciplines
- Decision-Making under Uncertainty
  - Decision-Making under Strict Uncertainty
    - Decision Table
    - Various Decision Criteria
  - Decision-Making under Risk
    - Expected Value
    - Sensitivity Analysis

Decision Analysis

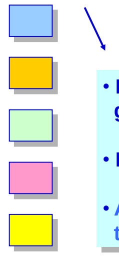
Decision Analysis: Applied Decision Theory

Provides a framework for analyzing decision problems by
- structuring and breaking them down into more manageable parts,
- explicitly considering the:
  - possible alternatives,
  - available information
  - uncertainties involved, and
  - relevant preferences
- combining these to arrive at optimal (or "good") decisions

Decision-Making Problem

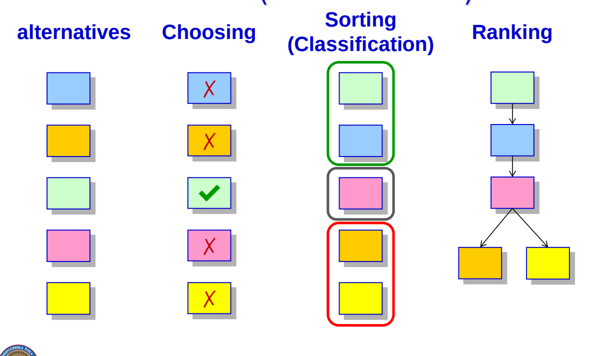
alternatives (options)

goals (objectives)

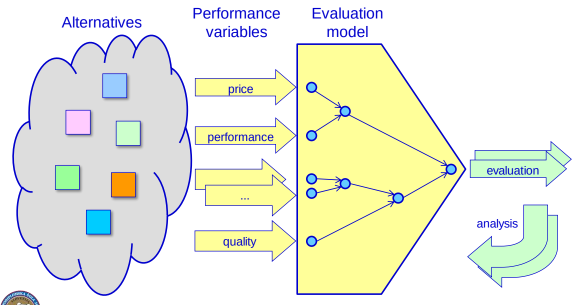


- **FIND the option that best satisfies the goals**
- **RANK options according to the goals**
- **ANALYSE, JUSTIFY, EXPLAIN, …, the decision**

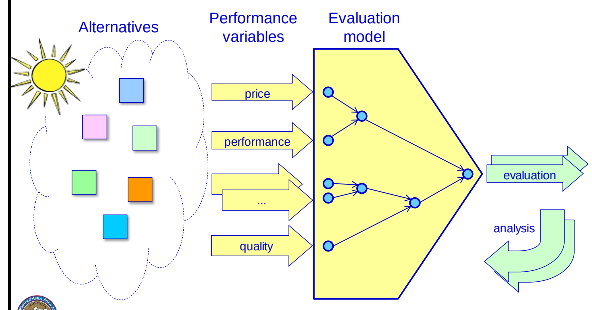
Decision Tasks ("Problematics")



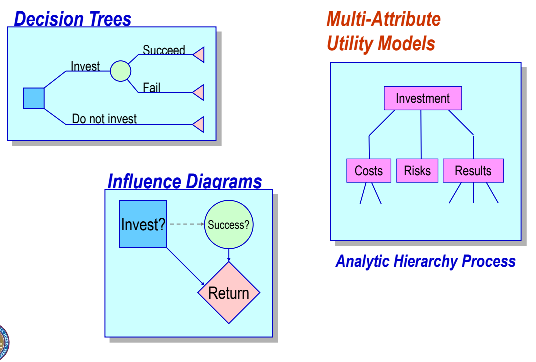
Evaluation Models



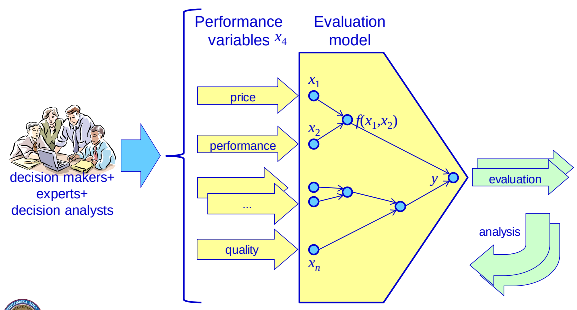
Evaluation Models



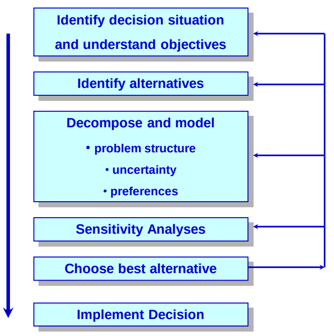
Types of Models in Decision Analysis



Evaluation Models



The Decision Analysis Process



Decision-Making under Uncertainty

Decision-Making Problem

Suppose that one must choose between several *uncertain alternatives*.

Given:

- *Alternatives*;
- The *consequences* of choosing each alternative, described with a *single* number, e.g. profit / loss in € or aggregated value.

Task: Which alternative to choose?

Decision Table

Decision-Making under Strict Uncertainty

State of the world (Event) θ	Value of alternatives $1 \ldots m$: a_1	...	a_m
θ_1	y_{11}	...	y_{1m}
:	:		:
θ_n	y_{n1}	...	y_{nm}

Decision-Making under Risk

State of the world (Event) θ	Probability that θ will happen $P(\theta)$	Value of alternatives $1 \ldots m$: a_1	...	a_m
θ_1	$p(\theta_1)$	y_{11}	...	y_{1m}
:	:	:		:
θ_n	$p(\theta_n)$	y_{n1}	...	y_{nm}

Working Example

A manufacturing company, faced with a possible increase in demand for its product, considers the following:

Alternatives:

1. *status quo*: no change
2. *extend*: extending their production line buying a new machine
3. *build*: building a new production hall with new equipment
4. *cooperate*: finding additional business parters for production

Uncertainty involved:
Market reaction: after the decision, the sales can *increase* or *decrease*.

Consequences:
Expected profit, shown in decision table on the next slide

Working Example

Decision table

		alternative			
		status quo	*extend*	*build*	*cooperate*
states	*decreased sales*	28	24	16	30
	increased sales	30	42	44	34

Decision-Making under Strict Uncertainty

Decision Criteria

- Dominance
- Pessimistic (Maximin, Wald's)
- Optimistic (Maximax)
- Hurwicz's
- Laplace's
- Minimax Regret

Dominance

- Choose the alternative with best consequences in all states of the world.
- Such alternative is seldom found.

		alternative			
		status quo	extend	build	cooperate
states	decreased sales	28	24	16	30
	increased sales	30	42	44	34

No dominant alternatives in this case

Pessimistic Criterion (Wald's, Maximin)

- Each alternative is represented by its *worst* possible consequence.
- According to these, the alternative with the *best* worst case is chosen.

		alternative			
		status quo	*extend*	*build*	*cooperate*
states	*decreased sales*	*28*	*24*	*16*	*30*
	increased sales	*30*	*42*	*44*	*34*
Pessimist		**28**	**24**	**16**	**30**

Optimistic Criterion (Maximax)

- Each alternative is represented by its *best* possible consequence.
- The alternative for which this *best* consequence is best is chosen.

		alternative			
		status quo	*extend*	*build*	*cooperate*
states	*decreased sales*	*28*	*24*	*16*	*30*
	increased sales	*30*	*42*	*44*	*34*
Optimist		**30**	**42**	**44**	**34**

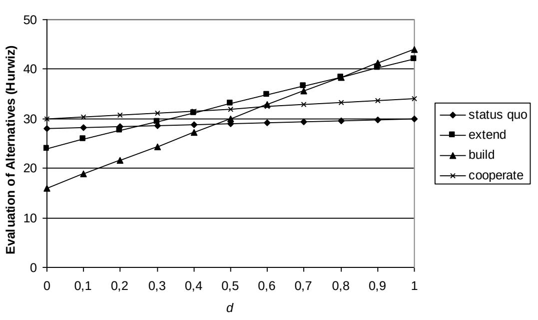
Hurwicz's Criterion

- Introduce a parameter $d \in [0,1]$.
- Combine Optimistic and Pessimistic criteria so that

$$u_h = d u_o + (1-d) u_p$$

		alternative			
		status quo	*extend*	*build*	*cooperate*
states	*decreased sales*	*28*	*24*	*16*	*30*
	increased sales	*30*	*42*	*44*	*34*
Pessimist		**28**	**24**	**16**	**30**
Optimist		**30**	**42**	**44**	**34**
Hurwiz (*d*=0,3)		**28,6**	**29,4**	**24,4**	**31,2**

Hurwicz's Criterion



Laplace's Criterion

- Consider all states (events) equally likely,
- thus, consider the *average* of outcomes for each alternative.

		alternative			
		status quo	*extend*	*build*	*cooperate*
states	*decreased sales*	*28*	*24*	*16*	*30*
	increased sales	*30*	*42*	*44*	*34*
Laplace		**29**	**33**	**30**	**32**

Minimax Regret

The **regret** r_{ij} for the alternative a_j in state θ_i is equal to the difference between the best alternative in given state θ_i and a_j: $r_{ij} = \max_k(y_{ik}) - y_{ij}$

Choose the alternative having the least maximum regret.

		alternative			
		status quo	*extend*	*build*	*cooperate*
States	*decreased sales*	30–28=2	30–24=6	30–16=14	30–30=0
	increased sales	44–30=14	44–42=2	44–44=0	44–34=10
Regret		**14**	**6**	**14**	**10**

Summary

		alternative			
		status quo	*extend*	*build*	*cooperate*
states	*decreased sales*	*28*	*24*	*16*	*30*
	increased sales	*30*	*42*	*44*	*34*
Pessimist		**28**	**24**	**16**	**30**
Optimist		**30**	**42**	**44**	**34**
Hurwiz (*d*=0,3)		**28,6**	**29,4**	**24,4**	**31,2**
Laplace		**29**	**33**	**30**	**32**
Regret		**14**	**6**	**14**	**10**

Questions

- If you were the manager, which alternative would you take? Why?
- Is this really the best alternative? Why? Under which circumstances it is best?
- What can you say about the *status quo* alternative? According to the analysis, when should be it taken, or should it be taken at all?

Questions

Assess the presented decision criteria:

- Describe the prevalent characteristics of each criterion
- What do you think about the criteria:
  - Are they comprehensible?
  - Are they realistic?
  - Are they useful for practice?
  - Which is your favourite criterion?
- Is there a single "best" criterion? Which and why?

Decision-Making under Risk

Working Example

Now we know (or estimate) the *probablity* of states

		alternatives			
states	***probability***	*status quo*	*extend*	*build*	*cooperate*
decreased sales	25%	28	24	16	30
increased sales	75%	30	42	44	34

Decision Criteria

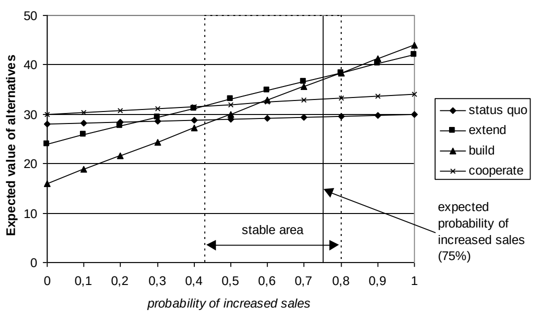
- Mode: Select the most probable state
- Expected Value (EV), Expected Monetary Value (EMV)

Expected (Monetary) Value

Maximise the expected value: $EV_i = \sum_{j=1}^{n} p(\theta_j) y_{ji}$

		alternatives			
states	**probability**	*status quo*	*extend*	*build*	*cooperate*
decreased sales	25%	28	24	16	30
increased sales	75%	30	42	44	34
Expected value		0,25×28+ 0,75×30= **29,5**	0,25×24+ 0,75×42= **37,5**	0,25×16+ 0,75×44= **37**	0,25×30+ 0,75×34= **33**

Sensitivity Analysis



Exercise 1

	P(θ)	a_1	a_2	a_3
θ_1	2/9	8	4	20
θ_2	3/9	7	15	10
θ_3	4/9	6	5	0

Given this decision table:

- Determine which alternative is best according to all the criteria (Dominance, Pessimistic, Optimistic, Hurwiz (d=0.7), Laplace, Regret, Mode, Expected Value).
- Draw a chart evaluating the Hurwiz's criterion for $d \in [0,1]$.
- Do sensitivity analysis.

Exercise 2

Help the farmer who is deciding which crop to plant in the face of uncertain weather and resulting crop yield:

probability	Weather .55 Normal	.15 Drought	30 Rainy
Plant soybeans	$ 10	5	12
Plant corn	7	8	13

profit per acre

Exercise 3

1. Define a decision problem of your own,
2. represent it in a decision table,
3. and repeat the steps of Exercise 1

Exercise 4

Using some decision table,
implement in spreadsheet software (such as MS Excel):

- evaluation of alternatives using all the criteria,
- drawing the chart associated with Hurwiz's criterion
- drawing the sensitivity analysis chart

Compare the two charts.

Decision Analysis Part 2: Decision Trees

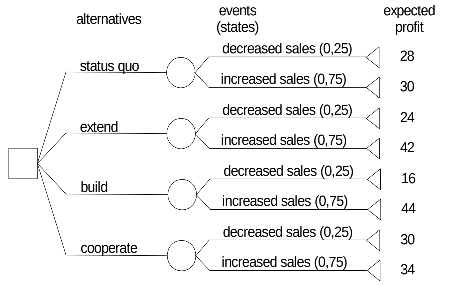
Working Example

Decision table (Payoff matrix)

		alternative			
		status quo	*extend*	*build*	*cooperate*
states	*decreased sales*	28	24	16	30
	increased sales	30	42	44	34

Working Example

Equivalent *decision tree*:



Decision Tree

Different from decision trees used in Machine Learning:

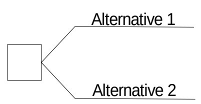
- different types of nodes
- always drawn horizontally, from left to right
- "hand-crafted", not learned from data

Decision tree represents the decision problem in terms of chains of consecutive *decisions* and chance *events*.

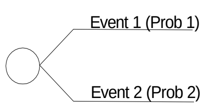
Time proceeds from *left to right*.

Uncertainties associated with chance events are modelled by *probabilities*.

Components of Decision Trees



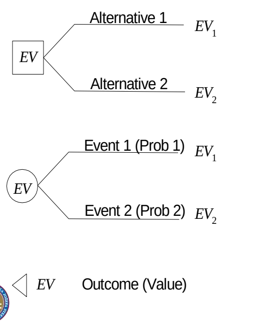
Decision node:
represents alternatives



Chance Node:
represents events (states of nature)

Terminal (End) Node:
represents consequences of decisions

Solving Decision Trees



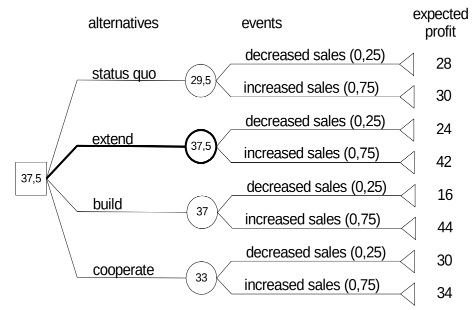
From right to left:

$EV = \max_i EV_i$ [maximize profit]
or
$EV = \min_i EV_i$ [minimise losses]

$EV = \sum_i p_i EV_i$

$EV = \text{Value}$

Solved Decision Tree



Decision Tree Development

1. Place decision and chance nodes in a logical time order
2. Independent chance nodes can be placed in any order
3. Estimate probabilities of all chance events
4. The sum of probabilities in a chance node must be 1
5. In terminal nodes, specify consequences by a single performance measure, e.g.:
   - money,
   - aggregate utility or
   - results of a multiple criteria analysis

Common Mistakes

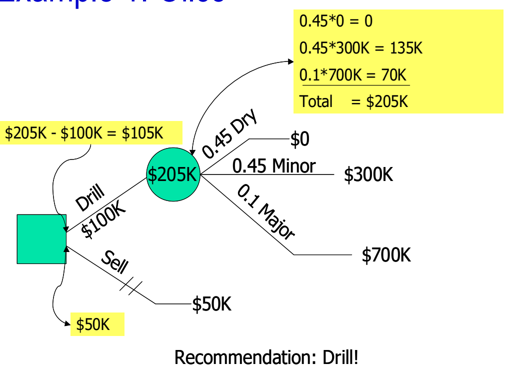
1. Decision and chance nodes are in wrong order: Only chance nodes whose results are known at the time of decision can precede a decision node
2. Incorrect derivation of chance probabilities: Chance probabilities depend on each other and decisions made
3. Chance events with probability 0 can be left out
4. When solving the tree: Maximising instead of minimising, or vice versa

Example 1: Oilco

Mobon Oil Company has a lease on an offshore oil site. The lease is about to expire and they are faced with either developing the field or selling the lease to Excel Oil Co. for $50,000. It costs approximately $100,000 to drill a well. There is a 45% chance that the well is dry, a 45% chance that the well will have a minor strike and a 10% chance that they will have a major strike. For a typical minor strike the revenues average $300,000. If the strike is major the revenues average $700,000. What should Mobon do?

Source: http://www.cs.jmu.edu/common/coursedocs/isat411/Decision%20analyis%20lectures2a.ppt

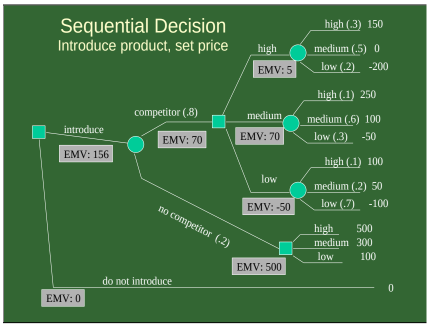
Example 1: Oilco



Recommendation: Drill!

Source: http://www.cs.jmu.edu/common/coursedocs/isat411/Decision%20analyis%20lectures2a.ppt

Example 2: Sequential Decision



Source: Decision Trees, http://academic.udayton.edu/CharlesEbeling/MSC%20535/Lectures/Lesson%202/Decision_Trees/decision%20trees.ppt

Other Important Concepts

1. Value of Perfect Information
2. Risk Profile

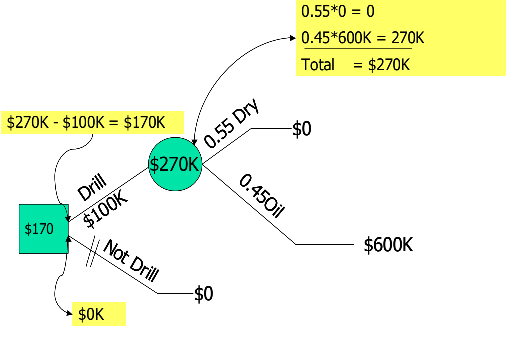
Value of Perfect Information

Example

- Oilco must determine whether of not to drill for oil in the South China Sea. It costs $100,000 to drill for oil and if oil is found the value of the oil is estimated to be $600,000. At present, Oilco believes there is a 45% chance that the field contains oil. What should Oilco do?
- What is the *value of perfect information* (knowledge of whether the field contains oil) to Oilco?

Source: http://www.cs.jmu.edu/common/coursedocs/isat411/Decision%20analyis%20lectures2a.ppt

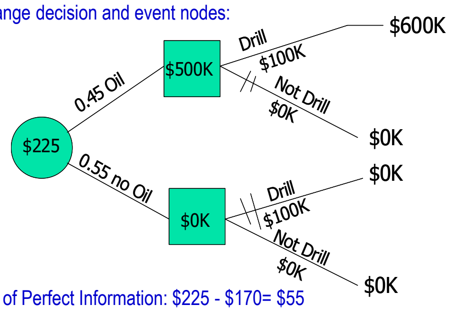
Ordinary Decision Tree



Recommendation: Drill!

Value of Perfect Information

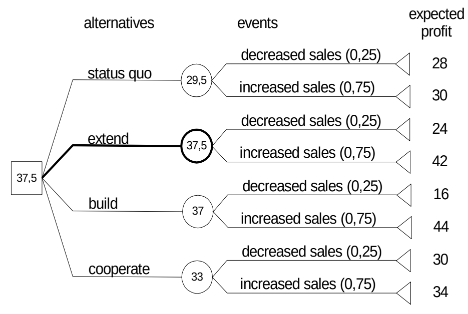
Exchange decision and event nodes:



Value of Perfect Information: $225 - $170= $55

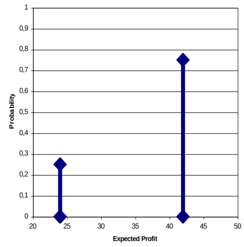
Risk Profile

Solved Decision Tree

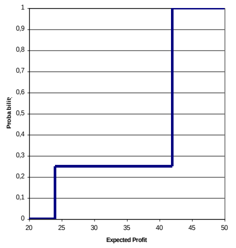


Risk Profile

Alternative: *extend*



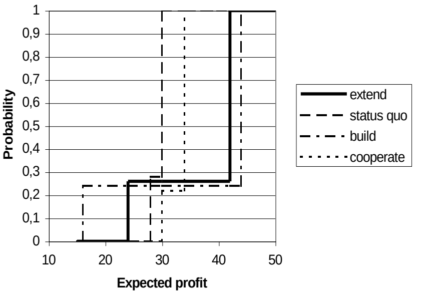
Probabilistic distribution



Cumulative distribution

Cumulative Risk Profile

All alternatives



Decision Tree Software

Decision Analysis Software

See http://kt.ijs.si/MarkoBohanec/dss.html

Decision Tree Software

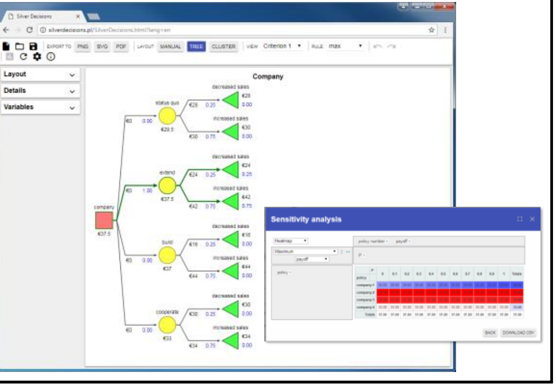
Add-Ins for Microsoft Excel:

- Simple Decision Tree: https://sites.google.com/site/simpledecisiontree/
- TreePlan: http://www.treeplan.com/
- PrecisionTree: http://www.palisade.com/precisiontree/

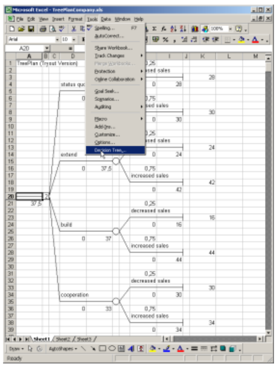
Decision-Tree Development Programs:

- SilverDecisions: http://silverdecisions.pl/
- TreeAge Pro (DATA): http://www.treeage.com/
- Vanguard Studio (DecisionPro): http://www.vanguardsw.com/products/vanguard-system/components/vanguard-studio/
- DPL: https://www.syncopation.com
- Insight Tree: http://www.insight-tree.com-about.com/

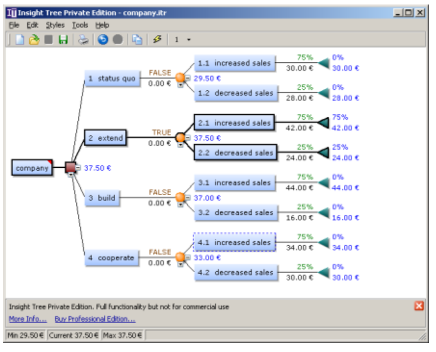
SilverDecisions



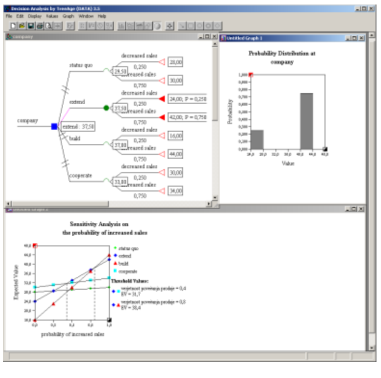
TreePlan



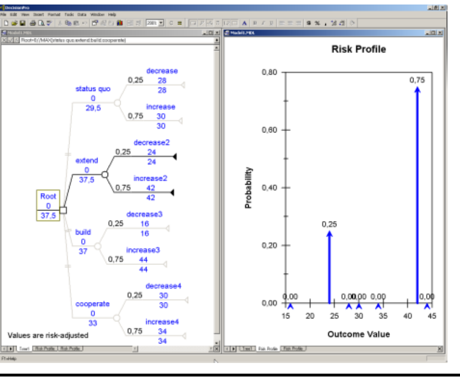
Insight Tree



DATA



DecisionPro



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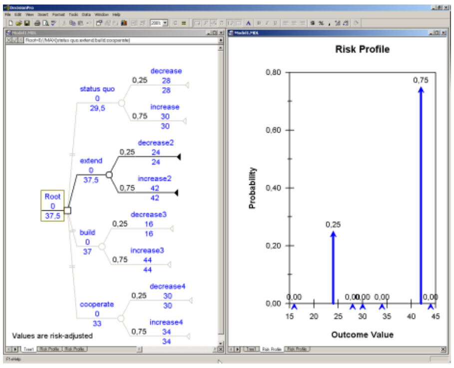
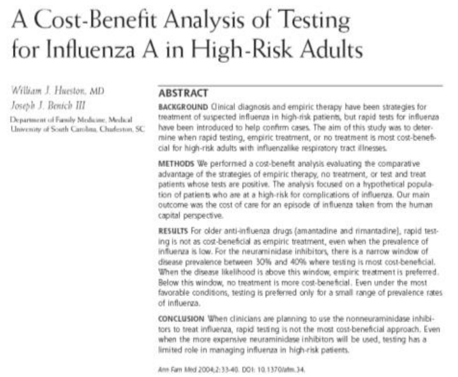


A Typical Application in Medicine



A Cost-Benefit Analysis of Testing for Influenza A in High-Risk Adults

William J. Hueston, MD
Joseph J. Benich III
Department of Family Medicine, Medical University of South Carolina, Charleston, SC

ABSTRACT

BACKGROUND Clinical diagnosis and empiric therapy have been strategies for treatment of suspected influenza in high-risk patients, but rapid tests for influenza have been introduced to help confirm cases. The aim of this study was to determine when rapid testing, empiric treatment, or no treatment is most cost-beneficial for high-risk adults with influenzalike respiratory tract illnesses.

METHODS We performed a cost-benefit analysis evaluating the comparative advantage of the strategies of empiric therapy, no treatment, or test and treat patients whose tests are positive. The analysis focused on a hypothetical population of patients who are at a high-risk for complications of influenza. Our main outcome was the cost of care for an episode of influenza taken from the human capital perspective.

RESULTS For older anti-influenza drugs (amantadine and rimantadine), rapid testing is not as cost-beneficial as empiric treatment, even when the prevalence of influenza is low. For the neuraminidase inhibitors, there is a narrow window of disease prevalence between 30% and 40% where testing is most cost-beneficial. When the disease likelihood is above this window, empiric treatment is preferred. Below this window, no treatment is more cost-beneficial. Even under the most favorable conditions, testing is preferred only for a small range of prevalence rates of influenza.

CONCLUSION When clinicians are planning to use the nonneuraminidase inhibitors to treat influenza, rapid testing is not the most cost-beneficial approach. Even when the more expensive neuraminidase inhibitors will be used, testing has a limited role in managing influenza in high-risk patients.

Ann Fam Med 2004;2:33-40. DOI: 10.1370/afm.34.

A Typical Application in Medicine

Figure 1. Decision tree used for the cost-benefit analysis.

Questions

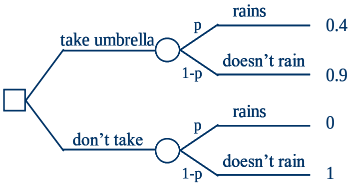
Compare decision tables with decision trees:
What do decision trees facilitate that decision tables don't?

Identify limitations and/or shortcomings of decision trees.

Identify types of decision problems suitable for the application of decision trees.

Exercise 1

Consider the decision tree:



1. Solve the tree for tomorrow's p
2. Do sensitivity analysis
3. Take the risk profile of your decision

Exercise 2

For the decision tree shown in the slide "Example 2: Sequential Decision" (Introduce Product):

1. do sensitivity anaysis with respect to *p*(*competitor*)
2. find the risk profile of alternative *introduce*.

Tree Development Exercise (1/3)

Service station problem:

- You are the owner of a service station on an intercity road. You have heard a rumour that the road may be upgraded or diverted along a different route. What do you do? What information will you need? How do you formulate a decision model?
- Think: before reading next slides, structure your own decision. You will need to specify an *objective*, identify *alternatives* available to you as the service station owner, and identify the *uncertainties* involved in this decision situation together with the possible *events*.

Adapted from: Making Decisions, http://www.imm.ecel.uwa.edu.au/unit450231/lectures/week%202.ppt

Tree Development Exercise (2/3)

Possible answers to the questions from the previous slide:

Objective: maximise the value of service station investment

Alternatives:

- *status quo*
- *sell*
- *extend*

Events:

- *unaltered* (p=0.5)
- *upgrade* (p=0.3)
- *divert* (p=0.2)

Tree Development Exercise (3/3)

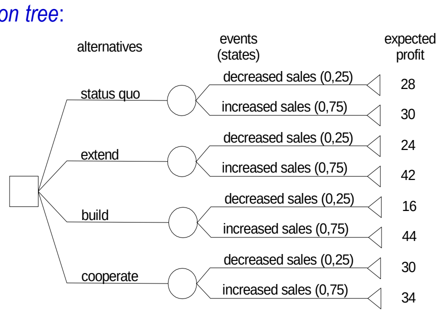
Proceed as follows:

1. Define decision table (include *consequences*)
2. Convert decision table to decision tree
3. Calculate EV and identify the best alternative
4. Do sensitivity analysis with respect to p(*unaltered*) [which problem do you encounter here?]
5. Find the risk profile of the best alternative

Decision Analysis Part 3: Influence Diagrams

Working Example

Decision tree:

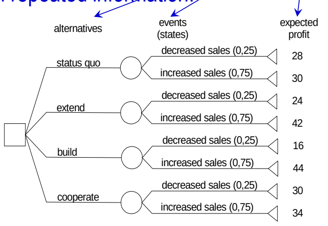


Motivation for Influence Diagrams

Decision trees:

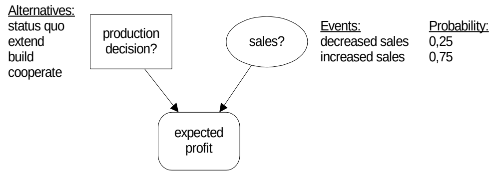
- sometimes too detailed,
- grow exponentially,
- contain repeated information.

Only three different elements:



Working Example

Equivalent *Influence diagram*:



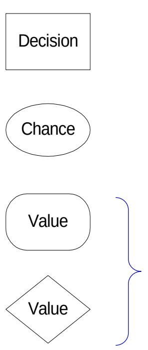
	status quo	extend	build	cooperate
decreased	28	24	16	30
increased	30	42	44	34

Influence Diagram

Influence diagram is a:
- high-level (compact),
- visual representation,
- displaying relationships between essential elements that affect the decision.

Two levels of detail:
- higher: only elements and relations
- lower: detailed information defined with each element

Elements of Influence Diagrams

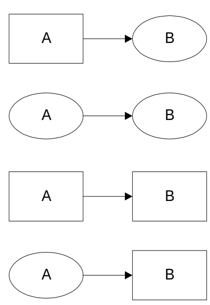


Decision node:
represents alternatives

Chance Node:
represents events (states of nature)

Value Node represents:
- consequences
- objectives, or
- calculations

Arcs in Influence Diagrams



Decision A affects the probabilities of event B;
Decision A is relevant for event B

The outcome of event A affects the probabilities of event B;
Event A is relevant for event B

Decison A occurs before decision B;
Decisions A and B are sequential

Decision B occurs after event A;
The outcome of A is known when deciding about B

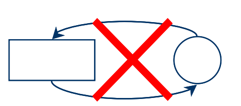
Developing Influence Diagrams

Two basic strategies:

- Start with outcomes and model towards decisions and events
- Gradually add more and more detail

Common Mistakes

1. An influence diagram is not a flowchart.
2. An arc from a chance node into a decision node means that the decision-maker knows the outcome of the chance node when making the decision.
3. There can be no cycles:



Decision Trees : Influence Diagrams

- DT display more information, the details of a problem, but they may become “messy”.
- ID show a general structure of a problem and hide details.
- ID are particularly valuable for the structuring phase of problem solving and for representing large problems.
- Solving algorithms: DT straightforward, ID difficult
- Any properly built ID can be converted into a DT, and vice versa.

- Bayesian networks are ID’s containing only event nodes

Solving Influence Diagrams

A. Convert ID to DT, solve DT

or

B. Solve directly by node reduction:
   1. Cleanup: one consequence C, no cycles, transform calculation nodes to one-event chance nodes...
   2. Repeat until ID solved:
      1. Reduce (calculate EV of) all chance nodes that directly precede C and do not precede any other node.
      2. Reduce (calculate EV of) the decision node that directly precedes C and has as predeccessors all of the other direct predeccessors of C.
   + arc reversal where there are no nodes corresponding to 2.2

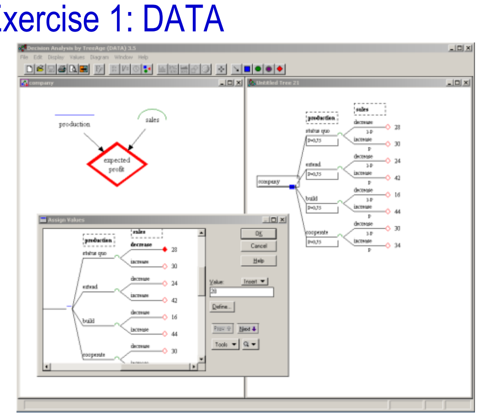
Influence Diagram Software

Add-Ins for Microsoft Excel:
- PrecisionTree: http://www.palisade.com

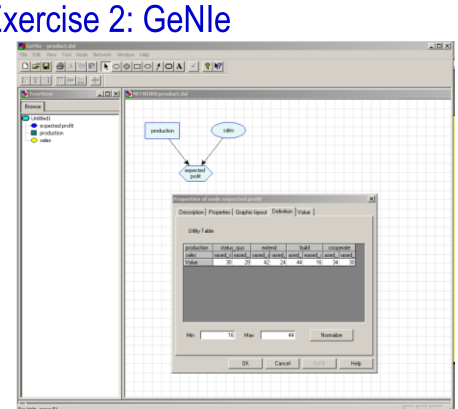
Influence-Diagram Development Programs:
- GeNIe: https://dslpitt.org/genie/
- TreeAge Pro (DATA): http://www.treeage.com/
- DPL: https://www.syncopation.com
- Analytica: http://www.lumina.com/ana/whatisanalytica.htm
- HUGIN: http://www.hugin.com/
- Netica: http://www.norsys.com/

Exercises

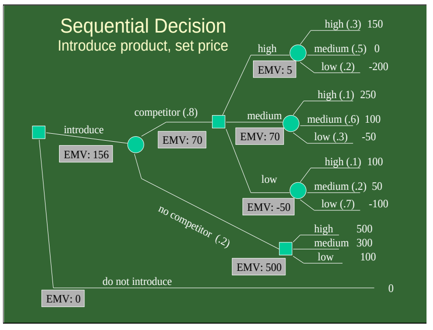
Exercise 1: DATA



Exercise 2: GeNIe

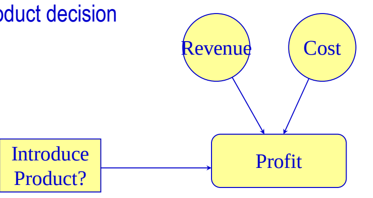


Exercise 3: Develop ID



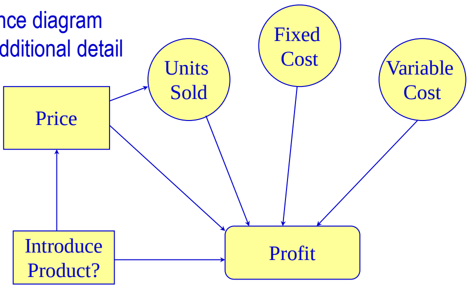
Example 1: Gradual Development (1/3)

Influence diagram of a new product decision

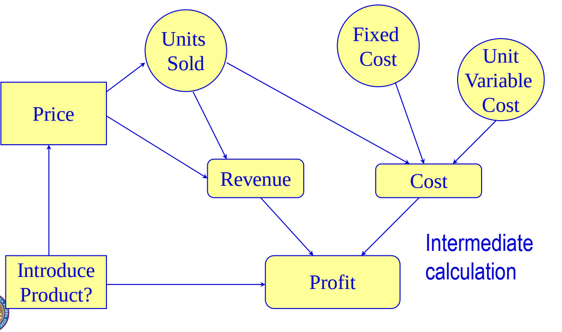


Example 1: Gradual Development (2/3)

Influence diagram with additional detail

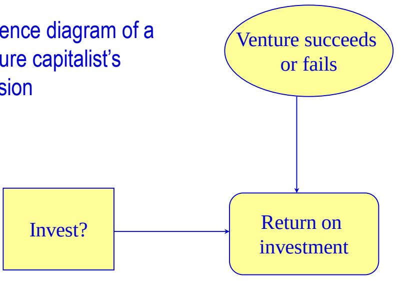


Example 1: Gradual Development (3/3)

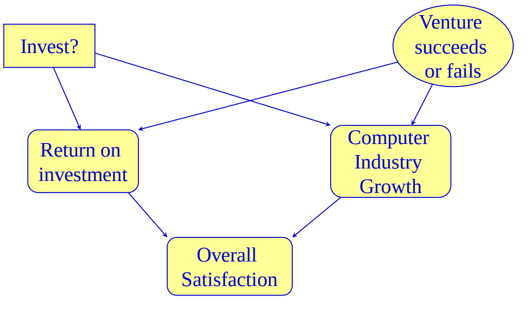


Example 2: Multiple Objectives (1/2)

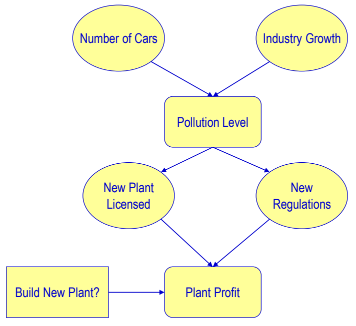
Influence diagram of a venture capitalist's decision



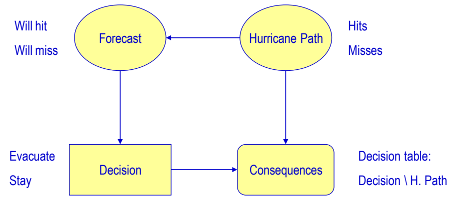
Example 2: Multiple Objectives (2/2)



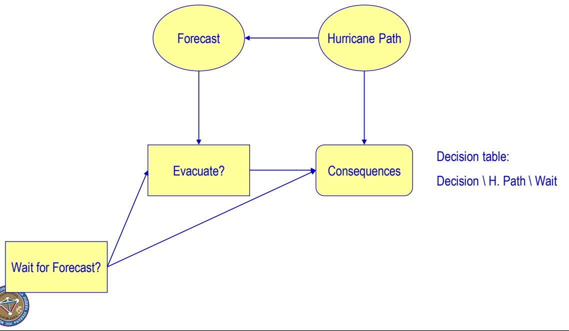
Example 3: Intermediate Calculations



Example 4: Evacuation Decision (1/2)



Example 4: Evacuation Decision (2/2)



Exercise 4

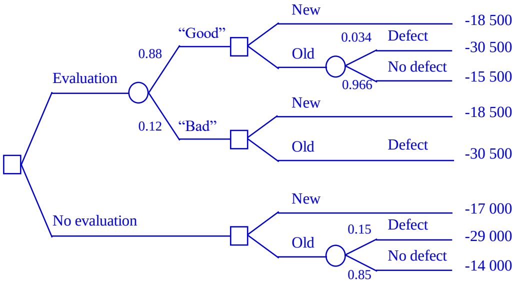
Create influence diagrams representing the decision trees encountered so far:

1. Oilco
2. Take an umbrella
3. Service station

Exercise 5: Tractor Buying (1/3)

- Your uncle is going to buy a tractor. He has two alternatives:
  1. A new tractor (17 000 €)
  2. An used tractor (14 000 €)
- The engine of the old tractor may be defect, which is hard to ascertain. Your uncle estimates a 15 % probability for the defect.
- If the engine is defect, he has to buy a new tractor and gets 2000 € for the old one.
- Before buying, your uncle can take the old tractor to a garage for an evaluation, which costs 1 500 €.
  - If the engine is OK, the garage can confirm it without exception.
  - If the engine is defect, there is a 20 % chance that the garage does not notice it.

Exercise 5: Tractor Buying (2/3)



Exercise 5: Tractor Buying (3/3)

Do the following:

1. Solve the decision tree
2. Develop equivalent influence diagram:
   1. structure of nodes
   2. detailed node data (names, values, probabilities)