**Decision Support: Multi-Attribute Methods**

Jožef Stefan International Postgraduate School, Ljubljana
Programme: Information and Communication Technologies (ICTs)
Course Web Page: [http://kt.ijs.si/MarkoBohanec/DS/DS.html/](http://kt.ijs.si/MarkoBohanec/DS/DS.html/)

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**Evaluation Models**

alternatives

EVALUATION

MODEL

EVALUATION

ANALYSIS

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**Multi-Attribute Models**

**Multi-Attribute Modelling: Why?**

- Systematic, structured approach (to difficult real-life problems)
- Model development:
  - problem decomposition into smaller, less-complex subproblems
  - requires understanding and careful elaboration of the problem
  - facilitates and motivates communication and knowledge interchange
- Analysis:
  - “what-if” analysis
  - sensitivity analysis
  - explanation:
    - how? (evaluation procedure)
    - why? (selective explanation of advantages/disadvantages)
- Contributes to better decisions:
  - understanding, justification, explanation, documentation

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**Multi-Attribute Model Structure**

**Multi-Attribute Model for Car Selection**

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**Quantitative Multi-Attribute Model for Car Selection**

Value Function

\[50 \times P_1 + 20 \times P_2 + 30 \times P_3\]

Aggregation Function

Marginal Value Functions

<table>
<thead>
<tr>
<th>ALTERNATIVES</th>
<th>VALUE</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. 22,000</td>
<td>6</td>
</tr>
<tr>
<td>2. 26,000</td>
<td>9</td>
</tr>
<tr>
<td>3. 19,000</td>
<td>8</td>
</tr>
</tbody>
</table>

**Qualitative Multi-Attribute Model for Car Selection**

Value Function (Decision Rules)

<table>
<thead>
<tr>
<th>CAR</th>
<th>PRICE</th>
<th>FUEL</th>
<th>SAFETY</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>high</td>
<td>high</td>
<td>unacc</td>
</tr>
<tr>
<td>2.</td>
<td>low</td>
<td>low</td>
<td>good</td>
</tr>
<tr>
<td>3.</td>
<td>low</td>
<td>med</td>
<td>acc</td>
</tr>
</tbody>
</table>

**Hierarchical Multi-Attribute Model**

Value (utility)

Value function

Aggregate attributes

Value function

Basic attributes

Alternative (option)

**Multi-Attribute Modelling: How?**

0. Problem identification
1. Tree (or hierarchy) of attributes
2. Utility functions
3. Evaluation and analysis of alternatives
4+ Implementation

**Exercise**

You would like to buy a new laptop computer for your own purposes (study, internet, fun, ...).

Suggest a suitable set of attributes and create a tree of attributes.

Consider the guidelines presented on the next two slides.
Developing Attribute Structure

Three basic strategies:

- **Top-Down**: Start with the overall evaluation (target objective), decompose it to sub-goals.
- **Bottom-Up**: Start with desirable characteristics, sub-goals. Group them into connected, meaningful sub-trees.
- **Middle-Out**: Combining the two above. Iteratively decompose (refine) and group (generalise) attributes.

Desirable features of attributes and their structure:

- **Completeness**: Do not overlook important attributes
- **Relevance (non-redundancy)**: Use only relevant attributes, omit redundant attributes
- **Minimality**: Use a minimal number of attributes
- **Orthogonality**: Basic attributes should be independent of each other
- **Operativity**: Basic attributes should be easy to assess or measure
- **Comprehensibility**: Create meaningful sub-trees of inter-related attributes

2. Utility Functions (Aggregation)

Aggregation: bottom-up aggregation of attributes’ values

```
CAR
- TECH CHAR
  - SAFETY
    - low, exc, unaccept
  - COMFORT
    - low, very low, unaccept
    - medi, accept, accept
    - high, good, exc
```

3. Evaluation and Analysis

- **direction**: bottom-up (terminal \(\rightarrow\) root attributes)
- **result**: each alternative evaluated

Interactive inspection
- “what-if” analysis
- sensitivity analysis
- explanation

MADM Tools

1. “Paper and Pencil” (Abacon)
2. Spreadsheets and mathematical modelling software (MS Excel)
3. Specialized MADM software
One Thursday morning, Charles, instead of attending his Management Science Techniques for Consultants class, was mulling over his four job offers. His offers came from: Acme Manufacturing, Bankers Bank, Creative Consulting, and Dynamic Decision Making. He knew that factors such as location, salary, amount of management science (which he loved), and long term prospects were important to him, but he wanted some way to formalize the relative importance, and some way to evaluate each job offer.

### Working Example

#### Job Offers

<table>
<thead>
<tr>
<th>Attribute</th>
<th>Alternative A</th>
<th>Alternative B</th>
<th>Alternative C</th>
<th>Alternative D</th>
</tr>
</thead>
<tbody>
<tr>
<td>Location</td>
<td>2</td>
<td>6</td>
<td>7</td>
<td>4</td>
</tr>
<tr>
<td>Salary</td>
<td>1</td>
<td>9</td>
<td>1</td>
<td>5</td>
</tr>
<tr>
<td>Management Science</td>
<td>3</td>
<td>1</td>
<td>7</td>
<td>2</td>
</tr>
<tr>
<td>Long-Term Prospects</td>
<td>2</td>
<td>3</td>
<td>6</td>
<td>4</td>
</tr>
<tr>
<td>Total</td>
<td>68</td>
<td>110</td>
<td>131</td>
<td>84</td>
</tr>
</tbody>
</table>

#### Kepner-Tregoe Model

<table>
<thead>
<tr>
<th>Attribute</th>
<th>Weights</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Location</td>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>Salary</td>
<td>10</td>
<td>1</td>
</tr>
<tr>
<td>Management Science</td>
<td>5</td>
<td>4</td>
</tr>
<tr>
<td>Long-Term Prospects</td>
<td>3</td>
<td>1</td>
</tr>
</tbody>
</table>

#### Kepner-Tregoe: What-If Analysis

<table>
<thead>
<tr>
<th>Attribute</th>
<th>Alternative A</th>
<th>Alternative B</th>
<th>Alternative C</th>
<th>Alternative D</th>
</tr>
</thead>
<tbody>
<tr>
<td>Location</td>
<td>2</td>
<td>6</td>
<td>7</td>
<td>4</td>
</tr>
<tr>
<td>Salary</td>
<td>1</td>
<td>9</td>
<td>1</td>
<td>5</td>
</tr>
<tr>
<td>Management Science</td>
<td>3</td>
<td>1</td>
<td>7</td>
<td>2</td>
</tr>
<tr>
<td>Long-Term Prospects</td>
<td>2</td>
<td>3</td>
<td>6</td>
<td>4</td>
</tr>
<tr>
<td>Total</td>
<td>131</td>
<td>106</td>
<td>129</td>
<td></td>
</tr>
</tbody>
</table>

### Kepner-Tregoe Model: Charts

#### Kepner-Tregoe: Sensitivity Analysis

Sensitivity analysis:

- **Value**
  - 0
  - 10
  - 20
  - 30
  - 40
  - 50
  - 60
  - 70
  - 80
  - 90
  - 100

- **Weight of salary**
  - 0
  - 2
  - 4
  - 6
  - 8
  - 10
  - 12
  - 14
  - 16
  - 18
  - 20

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Marko Bohanec: DS Multi Attribute
**AHP**


**Characteristics:**
- based on multiple attribute hierarchies
- assessing weights by a pairwise comparison of attributes
- assessing preferences by a pairwise comparison of alternatives
- consistency analysis

**Pairwise Comparison Values**

1. Items i and j are of equal importance (preference)
2. Item i is weakly more important (better) than j
3. Item i is strongly more important (better) than j
4. Item i is very strongly more important (better) than j
5. Item i is absolutely more important (better) than j

2, 4, 6, 8 are intermediate values

**Assessing Weights**

1. Normalize the columns so that the sum equals 1
2. Take the average of rows.

<table>
<thead>
<tr>
<th>Location</th>
<th>Salary</th>
<th>MS</th>
<th>Long</th>
<th>Weights</th>
</tr>
</thead>
<tbody>
<tr>
<td>Location</td>
<td>0.091</td>
<td>0.103</td>
<td>0.091</td>
<td>0.059</td>
</tr>
<tr>
<td>Salary</td>
<td>0.455</td>
<td>0.513</td>
<td>0.545</td>
<td>0.471</td>
</tr>
<tr>
<td>MS</td>
<td>0.273</td>
<td>0.256</td>
<td>0.353</td>
<td>0.289</td>
</tr>
<tr>
<td>Long</td>
<td>0.182</td>
<td>0.128</td>
<td>0.353</td>
<td>0.130</td>
</tr>
</tbody>
</table>

**Assessing Preferences (Scores)**

For each attribute, e.g., Location, compare alternatives:

1. Normalize the columns so that the sum equals 1
2. Take the average of rows.

**Evaluation:**

\[ V(A) = \sum_{j=1}^{n} W_j V_j(A) \]

\( V(A) \) for each attribute:

- Acme: \( 0.174 \)
- Bankers: \( 0.239 \)
- Creative: \( 0.489 \)
- Dynamic: \( 0.044 \)

**Assessing Preferences (Scores)**

Scores for all the attributes:

<table>
<thead>
<tr>
<th>Location</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>0.161</td>
<td>0.137</td>
<td>0.175</td>
<td>0.227</td>
</tr>
<tr>
<td>B</td>
<td>0.323</td>
<td>0.275</td>
<td>0.257</td>
<td>0.318</td>
</tr>
<tr>
<td>C</td>
<td>0.484</td>
<td>0.549</td>
<td>0.514</td>
<td>0.423</td>
</tr>
<tr>
<td>D</td>
<td>0.032</td>
<td>0.039</td>
<td>0.057</td>
<td>0.045</td>
</tr>
</tbody>
</table>

**Evaluation:**

- Acme: \( 0.164 \)
- Bankers: \( 0.256 \)
- Creative: \( 0.385 \)
- Dynamic: \( 0.238 \)
Marko Bohanec: DS Multi Attribute

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DEX (Decision EXpert):

Method for qualitative multi-attribute modeling
DEX is similar to other multi-attribute methods:
1. Multiple attributes, hierarchically structured
2. Evaluation of alternatives: bottom-up aggregation

DEX (Decision EXpert)

Method for qualitative multi-attribute modeling
DEX is different from other multi-attribute methods:
1. Attributes are discrete, symbolic, qualitative

Multicriteria Modeling Software

<table>
<thead>
<tr>
<th>Software</th>
<th>Website</th>
</tr>
</thead>
<tbody>
<tr>
<td>Criterium DecisionPlus</td>
<td><a href="http://www.infoharvest.com/">http://www.infoharvest.com/</a></td>
</tr>
<tr>
<td>DECERNS</td>
<td><a href="http://www.duhs.slu.edu/product-decerns-model/">http://www.duhs.slu.edu/product-decerns-model/</a></td>
</tr>
<tr>
<td>AHP Software</td>
<td></td>
</tr>
<tr>
<td>Criterium DecisionPlus</td>
<td><a href="http://www.infoharvest.com/">http://www.infoharvest.com/</a></td>
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</tr>
<tr>
<td>Web-HIPRE Software</td>
<td></td>
</tr>
<tr>
<td>Web-HIPRE</td>
<td><a href="http://www.hipre.hut.fi/">http://www.hipre.hut.fi/</a></td>
</tr>
<tr>
<td>DEC (Decision EXpert):</td>
<td></td>
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<tr>
<td>Qualitative Multi-Attribute Modelling Method</td>
<td></td>
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DEX (Decision EXpert):

Method for qualitative multi-attribute modeling

DEX is similar to other multi-attribute methods:
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DEX (Decision EXpert)

Some Car

Buying, Maintain, Fuel, Safety, Comfort

DEX

Method for qualitative multi-attribute modeling
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DEC (Decision EXpert):

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Method for qualitative multi-attribute modeling
DEX is different from other multi-attribute methods:
1. Attributes are discrete, symbolic, qualitative

DEC

Numeric (quantitative):
BUYING = 12.233 €
numeric scale, e.g., 10

Symbolic (qualitative):
BUYING = medium
scale (high, medium, low)
**DEX Methodology**

Methodology

- Initial development
- DECMAK
- "Toolbox"

First applications

- HR and SW
- selection
- personnel management
- nursery schools

Selected applications

- Housing Fund
- Ministry of Science
- Technology and Industry
- health care
- medicine

Related

- HINT

Methodology

- Further improvement
- DECMAK
- Forced Improvement

International applications

- EU-projects
- agriculture, food, GMO
- project evaluation
- finance

Related

- model revision, proDEX

**DEX Evaluation**

A simple computer program for MADM that facilitates:

- Creation and editing of
  - model structure (tree of attributes)
  - value scales of attributes
  - decision rules (incl. using weights)
  - alternatives and their descriptions (data)
- Evaluation of alternatives (can handle missing values)
- Presentation of evaluation results with:
  - tables
  - charts
- Analyses: "what-if", "±1", selective explanation, comparison
- Preparing reports and charts

**DEXi Model**

Attribute Scale

- Location
- Salary
- MS (good, acc)
- Long (good, acc)

Tables

<table>
<thead>
<tr>
<th>Evaluation results</th>
<th>Job</th>
<th>Location</th>
<th>Salary</th>
<th>Satisfaction</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>unacc</td>
<td>unacc</td>
<td>unacc</td>
<td>unacc</td>
</tr>
<tr>
<td>B</td>
<td>acc</td>
<td>unacc</td>
<td>unacc</td>
<td>unacc</td>
</tr>
<tr>
<td>C</td>
<td>acc</td>
<td>acc</td>
<td>unacc</td>
<td>unacc</td>
</tr>
<tr>
<td>D</td>
<td>acc</td>
<td>acc</td>
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<td>B</td>
<td>acc</td>
<td>unacc</td>
<td>unacc</td>
<td>unacc</td>
</tr>
<tr>
<td>C</td>
<td>acc</td>
<td>acc</td>
<td>unacc</td>
<td>unacc</td>
</tr>
<tr>
<td>D</td>
<td>acc</td>
<td>acc</td>
<td>acc</td>
<td>acc</td>
</tr>
</tbody>
</table>

**DEXi Evaluation**

Evaluation results

- Location
- Salary
- Satisfaction
- MS (good, acc)
- Long (good, acc)

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Stages of MADM with DEXi

0. Problem Identification
   a. problem formulation
   b. formation of a decision-making group
   c. selection of decision-support methodology

1. Identification of Attributes
   a. unstructured list of attributes
   b. hierarchy (tree) of attributes
   c. measurement scales

2. Definition of Utility Functions (Decision Rules)

3. Evaluation and Analysis of Alternatives
   a. description of alternatives (data acquisition)
   b. evaluation of alternatives
   c. analysis

4. Implementation

1.a: Unstructured List of Attributes

Problem in Personnel Management: Select of a Candidate for a Job (e.g., a project manager)

- education
- age
- experience
- references
- knowledge
- work approach
- ability to work in a group
- health
- leadership
- organizational abilities
- loyalty
- intelligence
- communicativity
- character
- …

Do not overlook important attributes!

1.b: Tree of Attributes

Create meaningful, related groups
Avoid aggregate attributes having more than three descendants
1. c: Scales

Scales are discrete, typically ordered from bad to good.
Values should distinguish between importantly different characteristics.
Their number should gradually increase from bottom to the root.

2: Decision rules

Utility Functions, Bottom-Up Aggregation

3. a: Description of Alternatives

Candidate | Formal | For.Lang | Exper | Age | Comm | Leader | Test | Abilit |
---|---|---|---|---|---|---|---|---|
A | MSc | pas | to/year | 21-25 | good | more | B | |
B | PhD | act | more | 26-40 | aver | less | B | |
3.bc: Evaluation/Analysis of Alternatives

1. Evaluation
   • proceeds from bottom (basic attributes) to the root
   • result: qualitative evaluation of each alternative
   • handles missing (DEXi) or imprecise (DEX) alternative values

2. Analysis
   • interactive inspection of results
   • what-if analysis
     • compare alternatives
     • ±1 analysis
     • selective explanation
   • reports
   • charts

3.b: Evaluation of an Alternative

3.c: What-If Analysis

3.c: What-If Analysis
3.c: “±1” Analysis

3.c: Compare Alternatives

3.c: Selective Explanation

3.c: Selective Explanation

Charts and Reports

DEX and DEXi: Experience

- Wide applicability to various application areas
- Usually, solutions are specific (non-general)

1. Model development time
   - heavily problem-dependent: from hours to months
   - typical: 2 to 15 days

2. The most difficult stage
   - designing the tree of attributes

3. Appropriate decision problems
   - many attributes (> 15)
   - many alternatives (> 10)
   - prevailing qualitative decision-making, judgment
   - inaccurate or missing data
   - group decision making (communication and explanation)
   - sufficient resources available (expertise, time)
DEX and DEXi: Summary

1. Combination of multi-attribute decision making and expert systems

2. Characteristics:
   - qualitative (symbolic) decision making
   - explanation and analysis
   - active support in the acquisition of decision rules

3. Applicability:
   - for complex real-world problems
   - hundreds of real-life applications